CS686: Configuration Space I

Sung-Eui Yoon
(윤성의)

Course URL:
http://sglab.kaist.ac.kr/~sungeui/MPA

Announcements

- Make a project team of 2 or 3 persons for your final project
  - Each student has a clear role
  - Declare team members at KLMS by Apr-3; you don't need to define the topic by then
- Each student
  - Present two papers related to the project
  - 15 min ~ 20 min for each talk
- Each team
  - Give a mid-term review presentation for the project
  - Give the final project presentation

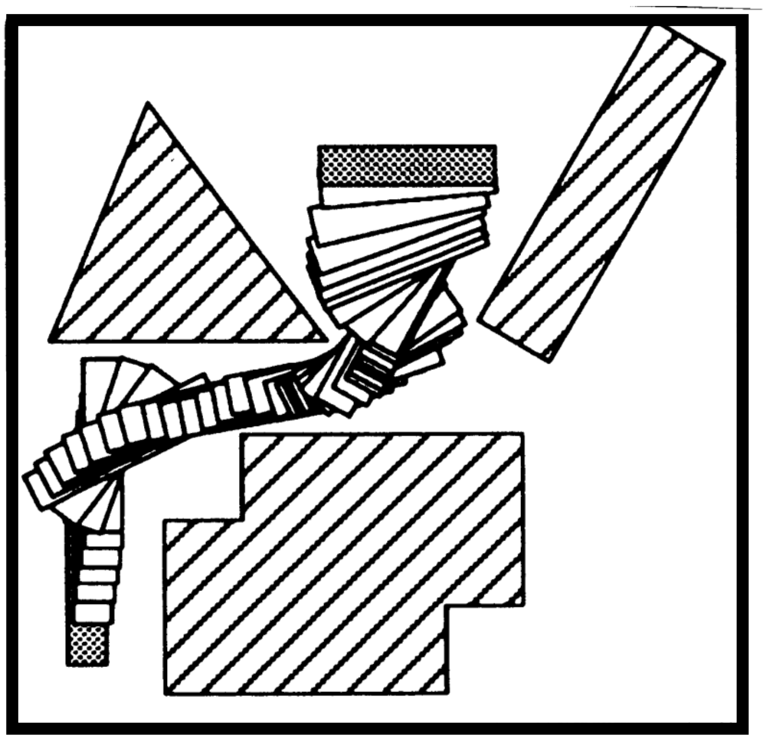
Tentative schedule

See the homepage

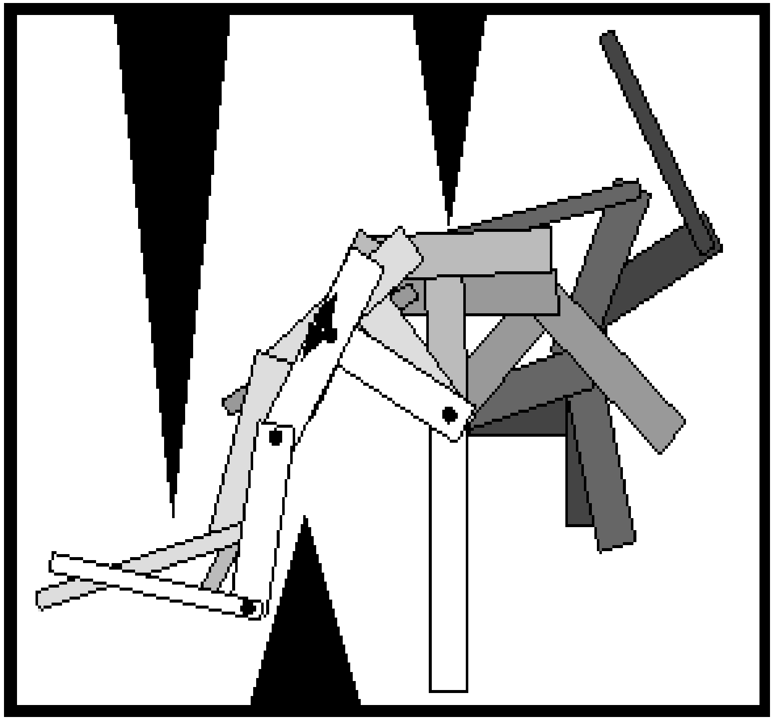
Class Objectives

- Configuration space
  - Definitions and examples
  - Obstacles
  - Paths
  - Metrics

What is a Path?



A box robot

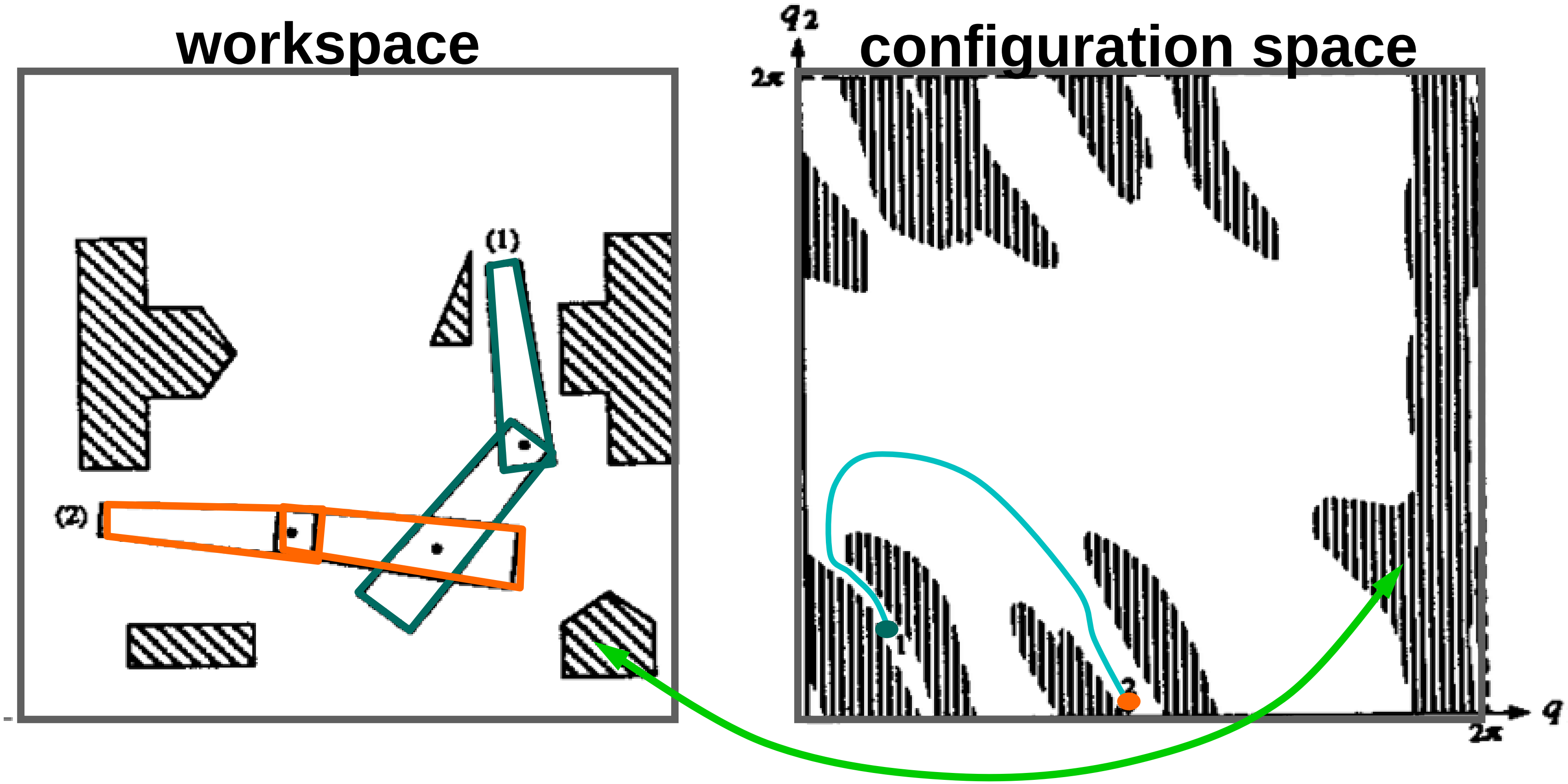


Linked robot

Rough Idea of C-Space

- **Convert rigid robots, articulated robots, *etc.* into points**
- **Apply algorithms in that space, in addition to the work space**

Mapping from the Workspace to the Configuration Space



Configuration Space

- Definitions and examples
- Obstacles
- Paths
- Metrics

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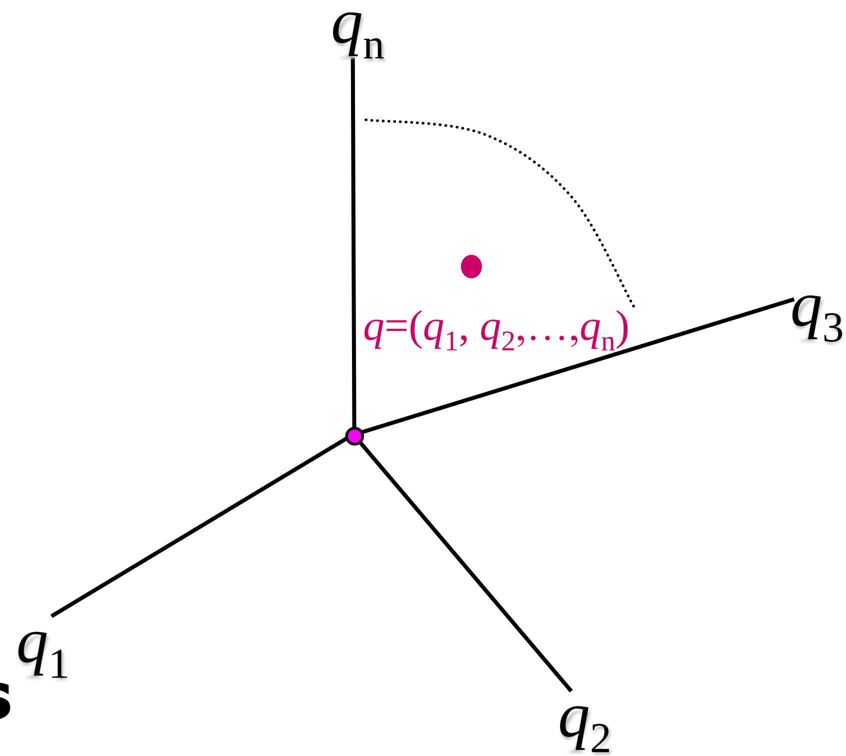
Configuration Space (C-space)

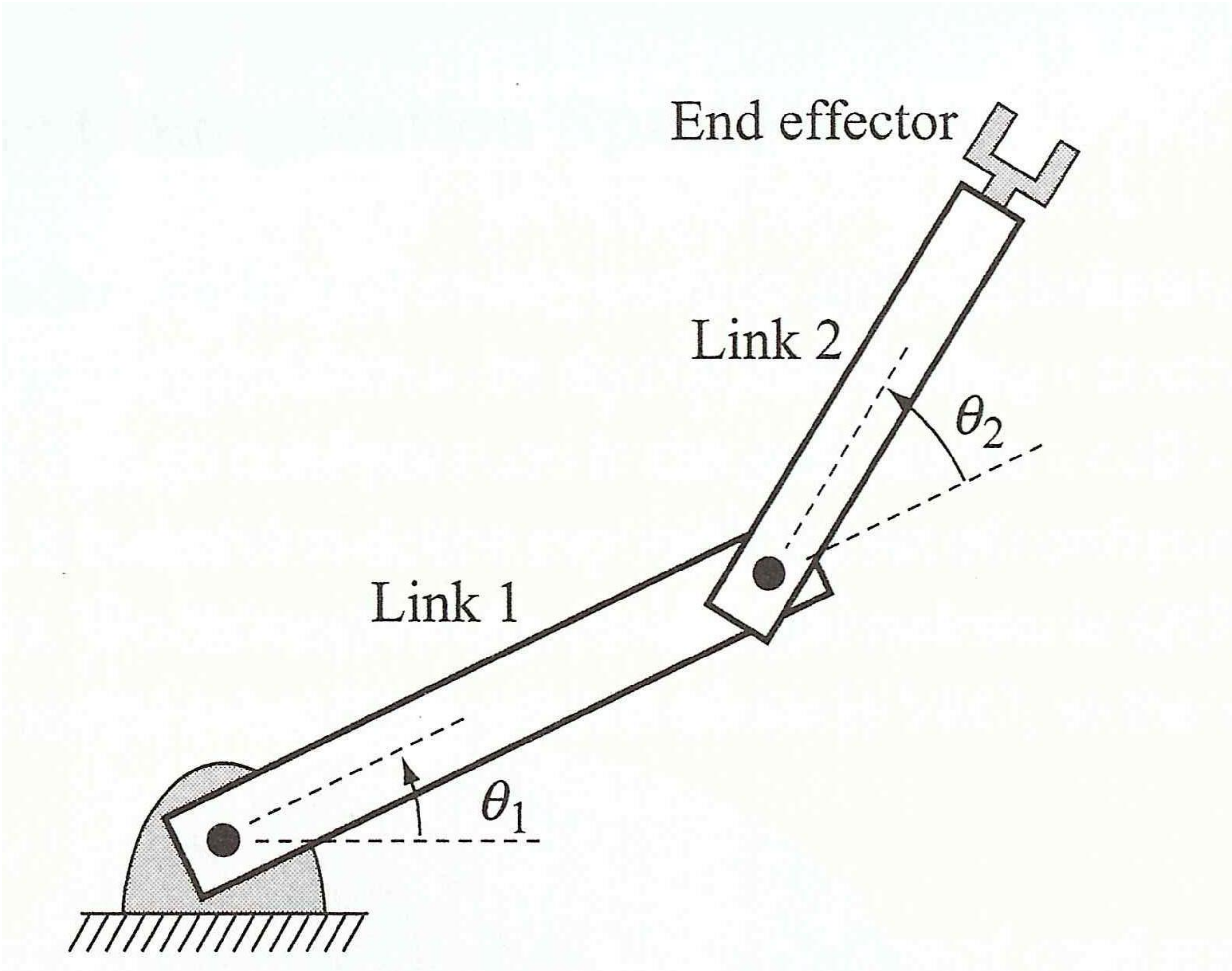
- The **configuration** of an object is a complete specification of the position of **every** point on the object
  - Usually a configuration is expressed as a vector of position & orientation parameters: $q = (q_1, q_2,\ldots,q_n)$



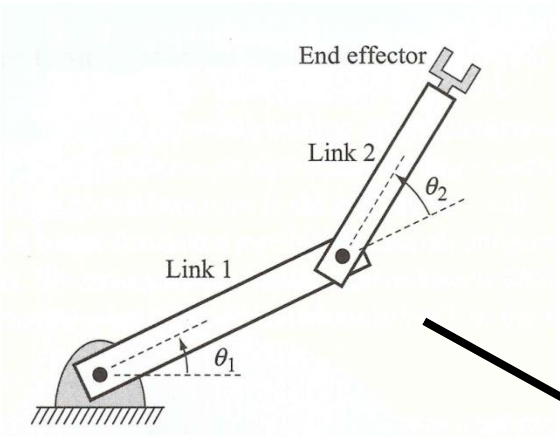
- The **configuration space** C is the set of all possible configurations
  - A configuration is a point in C

C-space formalism:
Lozano-Perez '79

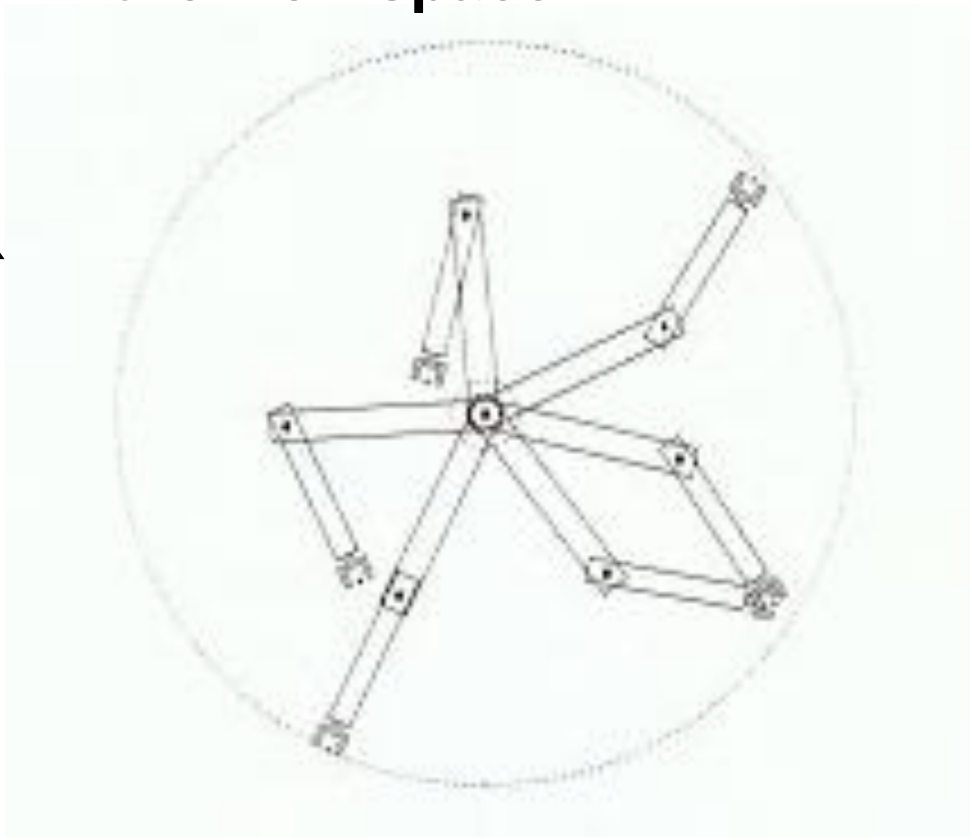
Examples of Configuration Spaces



Examples of Configuration Spaces

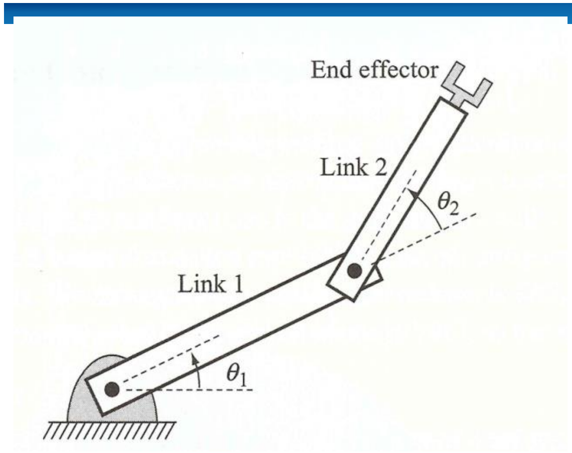


Consider the end-effector in the workspace?

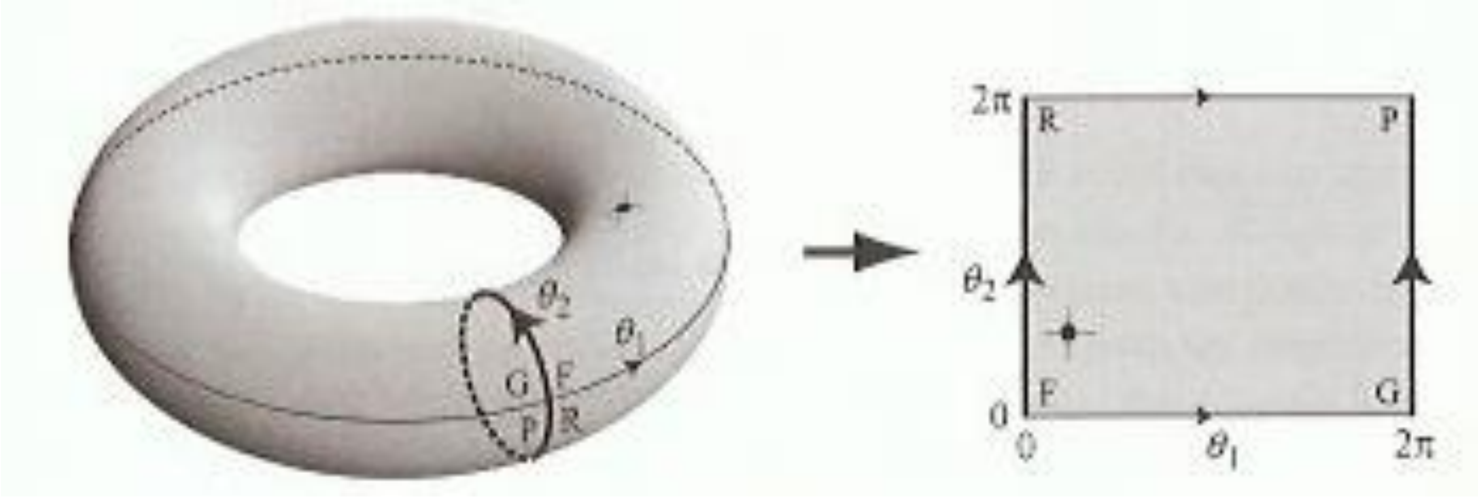


This is not a valid C-space!

Examples of Configuration Spaces

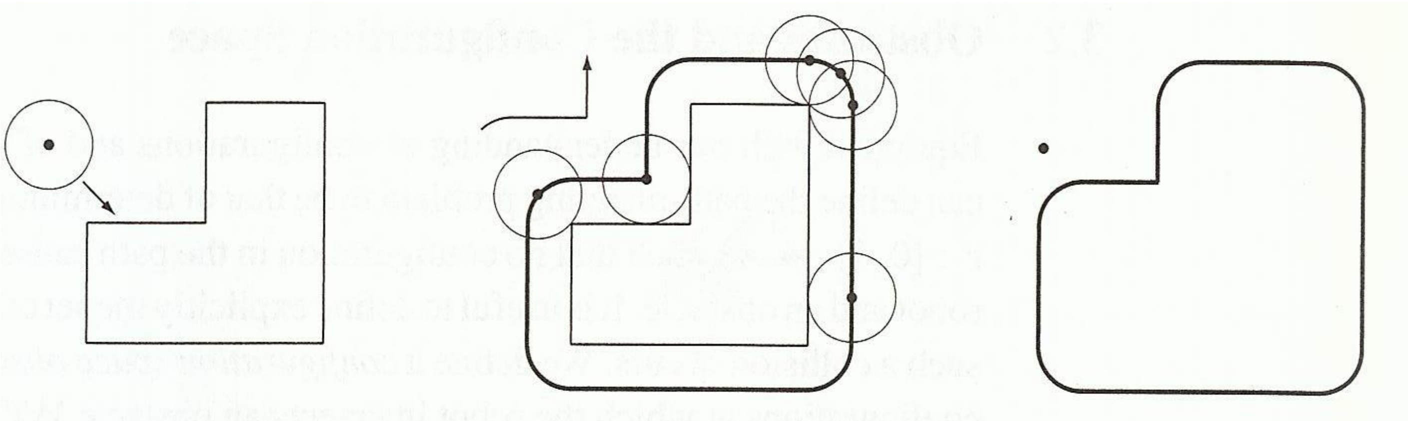


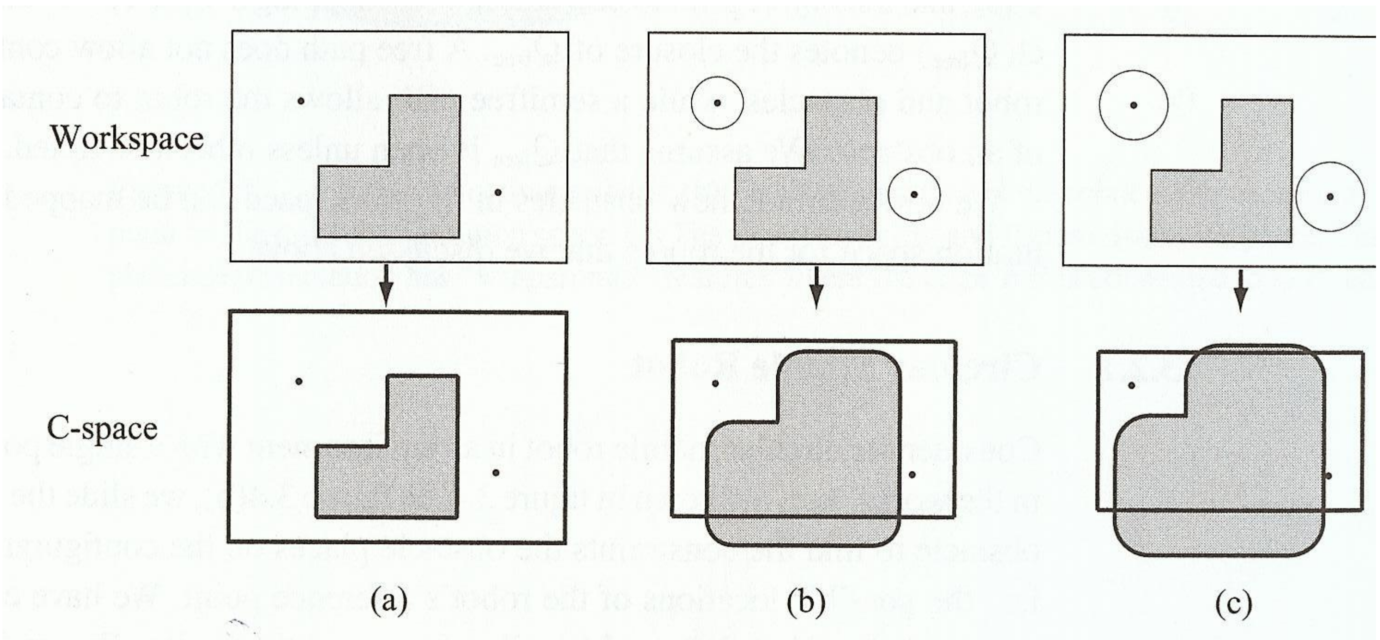
The topology of C is usually **not** that of a Cartesian space R^n.



$S^1 \times S^1 = T^2$

Examples of Circular Robot



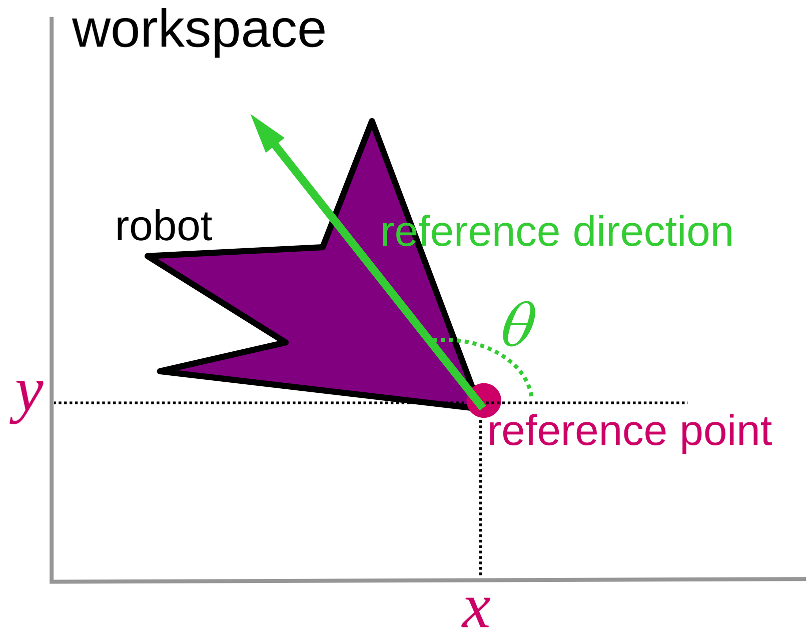


Dimension of Configuration Space

- The **dimension of the configuration space** is the **minimum** number of parameters needed to specify the configuration of the object completely

- It is also called the number of **degrees of freedom** (dofs) of a moving object

Example: Rigid Robot in 2-D Workspace



- **3-parameter specification:** $q = (x, y, \theta)$ with $\theta \in [0, 2\pi)$.
  - **3-D configuration space**

Example: Rigid Robot in 2-D workspace

- **4-parameter specification:** $q = (x, y, u, v)$ **with** $u^2+v^2 = 1$. **Note** $u = \cos\theta$ **and** $v = \sin\theta$

- **dim of configuration space =** 3
  - **Does the dimension of the configuration space (number of dofs) depend on the parametrization?**

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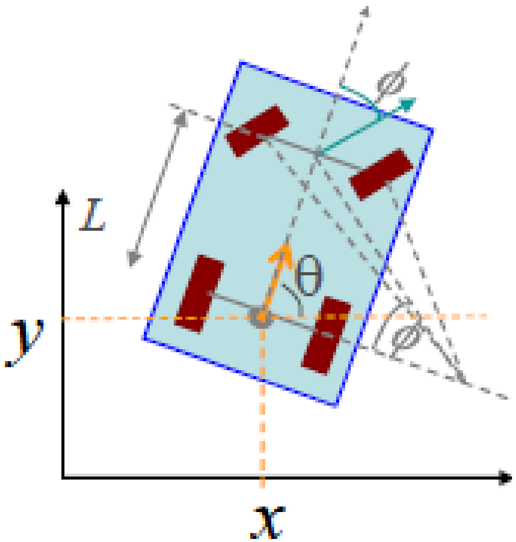
Holonomic and Non-Holonomic Contraints

- **Holonomic constraints**
  - $g(q, t) = 0$
- **Non-holonomic constraints**
  - $g(q, q', t) = 0$
  - This is related to the kinematics of robots
  - To accommodate this, the C-space is extended to include the position and its velocity
- **Dynamic constraints**
  - Dynamic equations are represented as $G(q, q', q'') = 0$
  - These constraints are reduced to non-holonomic ones when we use the extended C-space such as the state space

Example of Non-Holonomic Constraints

The path of the car is a curve tangent to its main rotation axis



$$dx \sin\theta - dy \cos\theta = 0$$

Computation of Dimension of C-Space

- **Suppose that we have a rigid body that can translate and rotate in 2D workspace**
  - **Start with three points: A, B, C (6D space)**

- **We have the following (holonomic) constraints**
  - **Given A, we know the dist to B: d(A,B) = |A-B|**
  - **Given A and B, we have similar equations: d(A,C) = |A-C|, d(B,C) = |B-C|**

- **Each holonomic constraint reduces one dim.**
  - **Not for non-holonomic constraint**

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Example: Rigid Robot in 3-D Workspace

- **We can represent the positions and orientations of such robots with matrices (i.e., SO (3) and SE (3))**

SO (n) and SE (n)

- **Special orthogonal group, SO(n), of n x n matrices R,**

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

that satisfy:

$r_{1i}^2 + r_{2i}^2 + r_{3i}^2 = 1$ **for all** i ,

$r_{1i}\, r_{1j} + r_{2i}\, r_{2j} + r_{3i}\, r_{3j} = 0$ **for all** $i \neq j$,

$\det(R) = +1$

Refer to the 3D Transformation at the undergraduate computer graphics.

- **Given the orientation matrix R of SO (n) and the position vector p, special Euclidean group, SE (n), is defined as:**

$$\begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

Example: Rigid Robot in 3-D Workspace

- q = (**position, orientation**) = (x, y, z, **???**)

- **Parametrization of orientations by matrix:**
  $q = (r_{11}, r_{12}, \ldots, r_{33}, r_{33})$ **where** $r_{11}, r_{12}, \ldots, r_{33}$ **are the elements of rotation matrix**

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \in SO(3)$$

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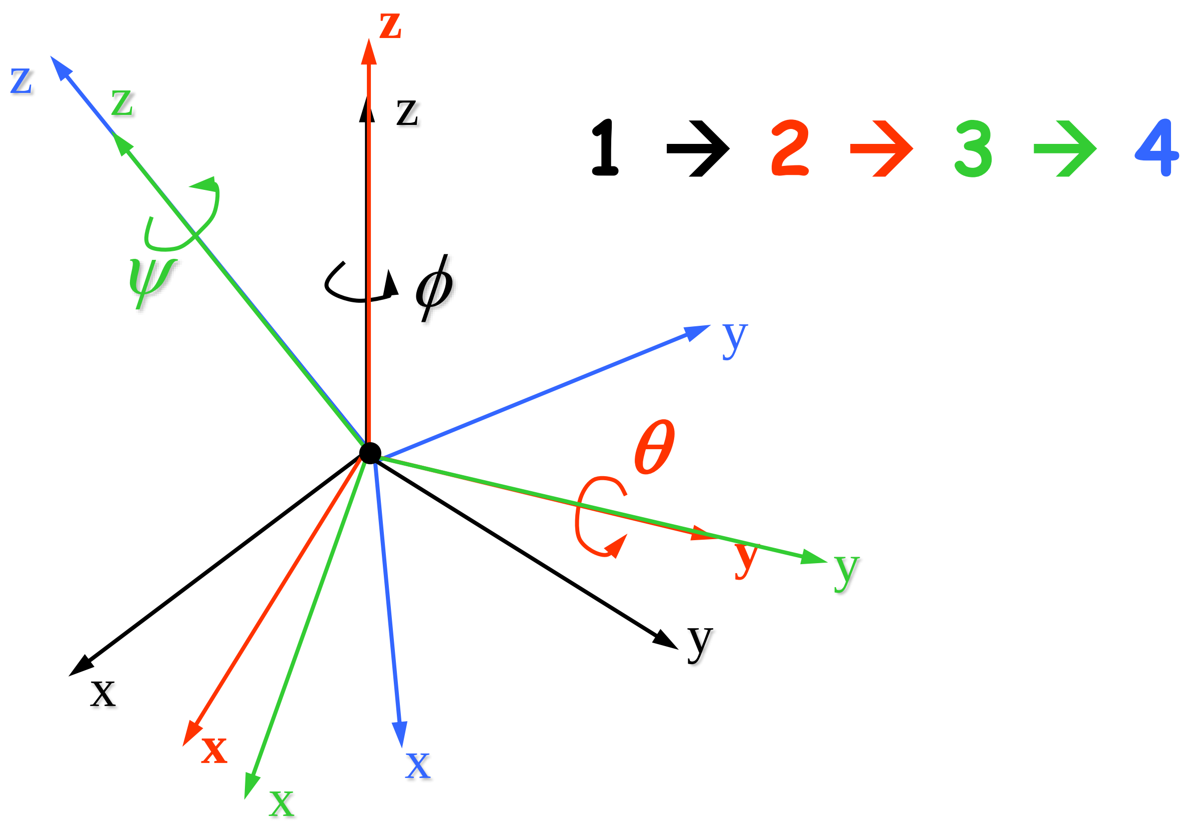
Example: Rigid Robot in 3-D Workspace

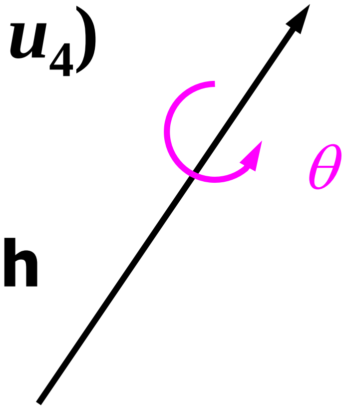
- **Parametrization of orientations by Euler angles:** (ϕ,θ,ψ)



Example: Rigid Robot in 3-D Workspace

- **Parametrization of orientations by unit quaternion:** $u = (u_1, u_2, u_3, u_4)$ **with** $u_1^2 + u_2^2 + u_3^2 + u_4^2 = 1.$
  - **Note** $(u_1, u_2, u_3, u_4) = (\cos\theta/2, n_x\sin\theta/2, n_y\sin\theta/2, n_z\sin\theta/2)$ **with** $n_x^2 + n_y^2 + n_z^2 = 1$
  - **Compare with representation of orientation in 2-D:** $(u_1, u_2) = (\cos\theta, \sin\theta)$

$\mathbf{n} = (n_x, n_y, n_z)$

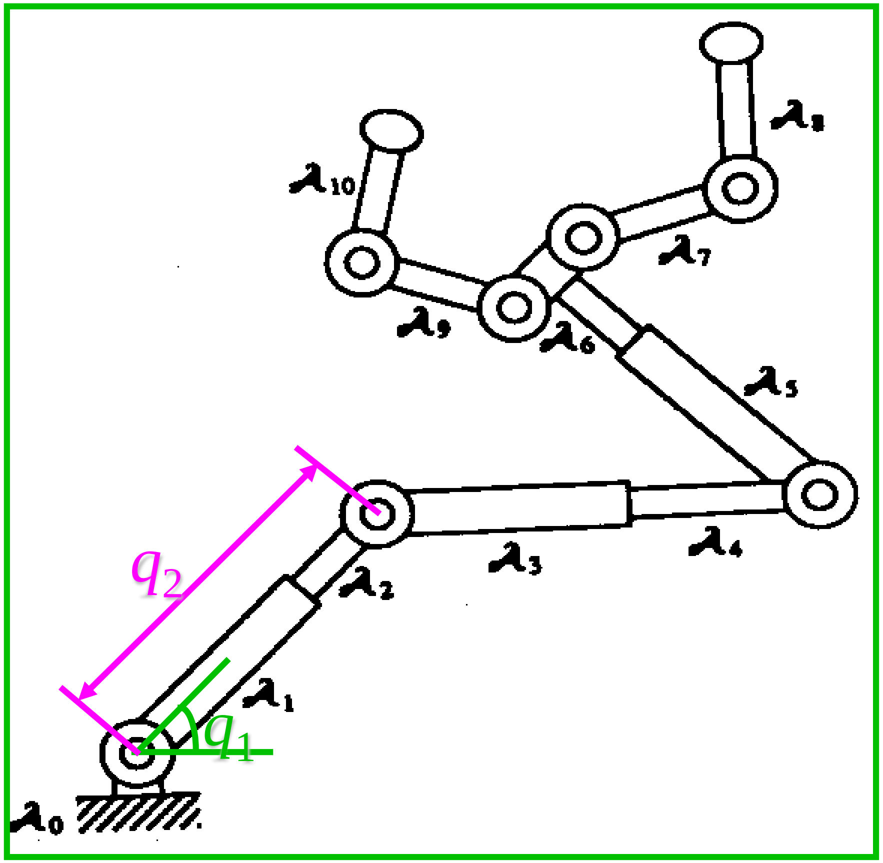


Example: Rigid Robot in 3-D Workspace

- **Advantage of unit quaternion representation**
  - **Compact**
  - **No singularity (no gimbal lock indicating two axises are aligned)**
  - **Naturally reflect the topology of the space of orientations**

- **Number of dofs = 6**
- **Topology:** $R^3 \times SO(3)$

Example: Articulated Robot



An articulated object is a set of rigid bodies connected at the joints.

- $q = (q_1, q_2, \ldots, q_{2n})$
- **Number of dofs** = $2n$
- **What is the topology?**

Class Objectives were:

- Configuration space
  - Definitions and examples
  - Obstacles
  - Paths
  - Metrics

Next Time….

- Configuration space
  - Definitions and examples
  - Obstacles
  - Paths
  - Metrics

Homework

- **Browse 2 ICRA/IROS/RSS/WAFR/TRO/IJRR papers**
  - **Prepare two summaries and submit at the beginning of every Tue. class, or**
  - **Submit it online before the Tue. Class**

- **Example of a summary (just a paragraph)**

  Title: XXX XXXX XXXX
  Conf./Journal Name: ICRA, 2016
  Summary: this paper is about accelerating the performance of collision detection. To achieve its goal, they design a new technique for reordering nodes, since by doing so, they can improve the coherence and thus improve the overall performance.

Homework for Every Class

- Go over the next lecture slides
- Come up with one question on what we have discussed today and submit at the end of the class
  - 1 for typical questions
  - 2 for questions with thoughts or that surprised me

- Write a question at least 4 times before the mid-term exam

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