Configuration Space I

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Course URL:
http://sglab.kaist.ac.kr/~sungeui/MPA

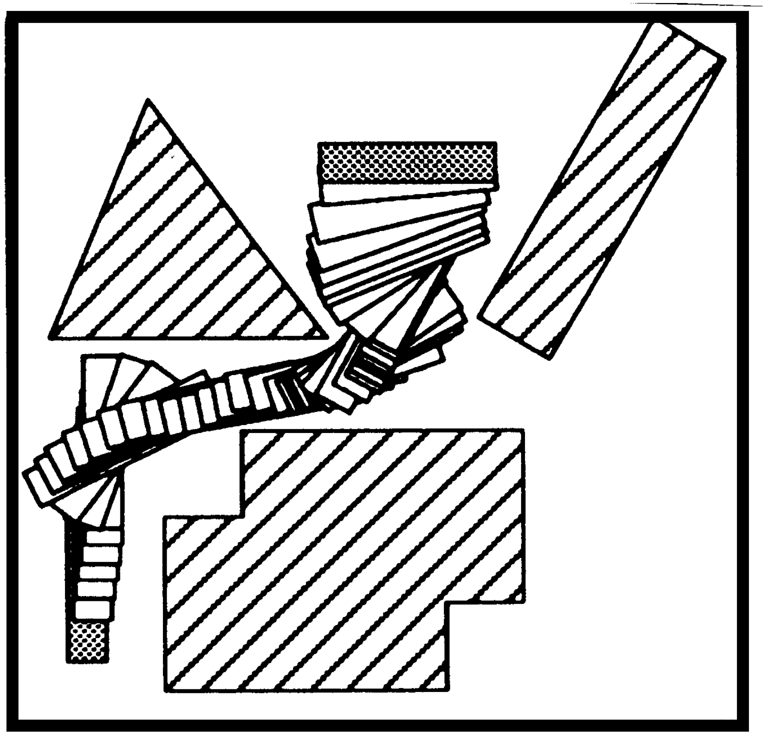
Announcements

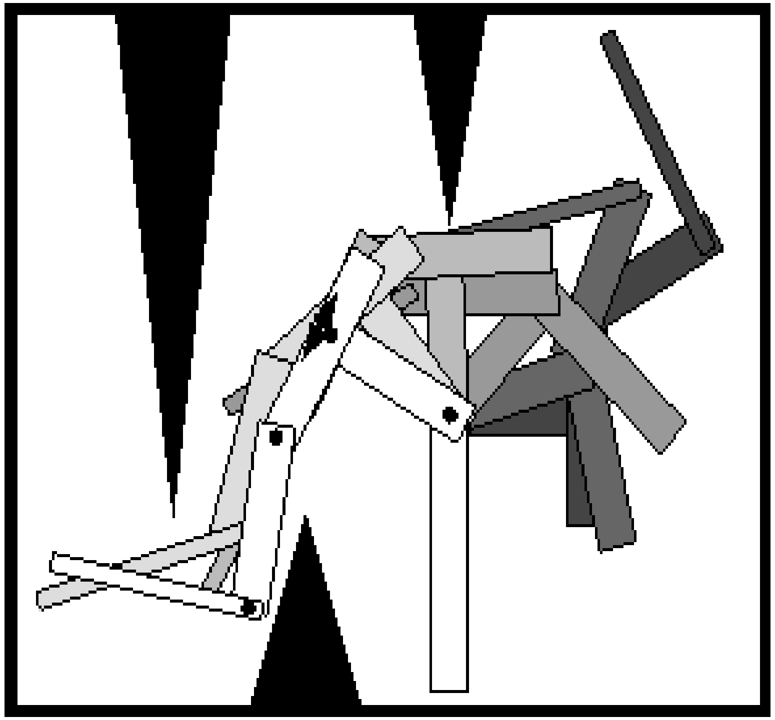
- Make a project team of two people for your final project
  - Each student has a clear role
  - Declare the team at the noah board soon
- Each student
  - Present two papers related to the project
- Each team
  - Give a mid-term review presentation for the project
  - Give the final project presentation

Class Objectives

- Configuration space
    - Definitions and examples
    - Obstacles
    - Paths
    - Metrics

What is a Path?





Rough Idea

- **Convert rigid robots, articulated robots, *etc.* into points**
- **Apply algorithms for moving points**

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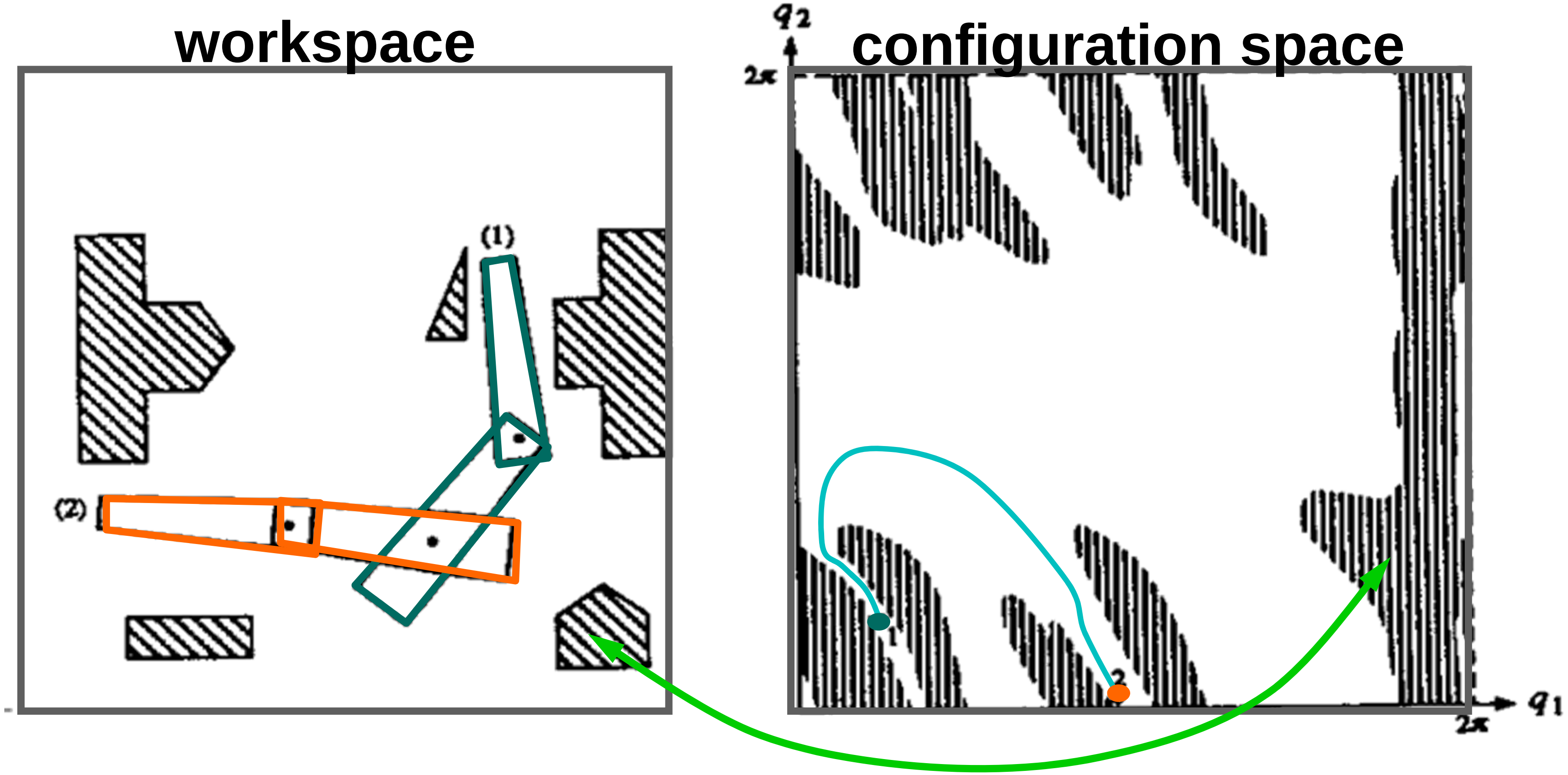
Mapping from the Workspace to the Configuration Space

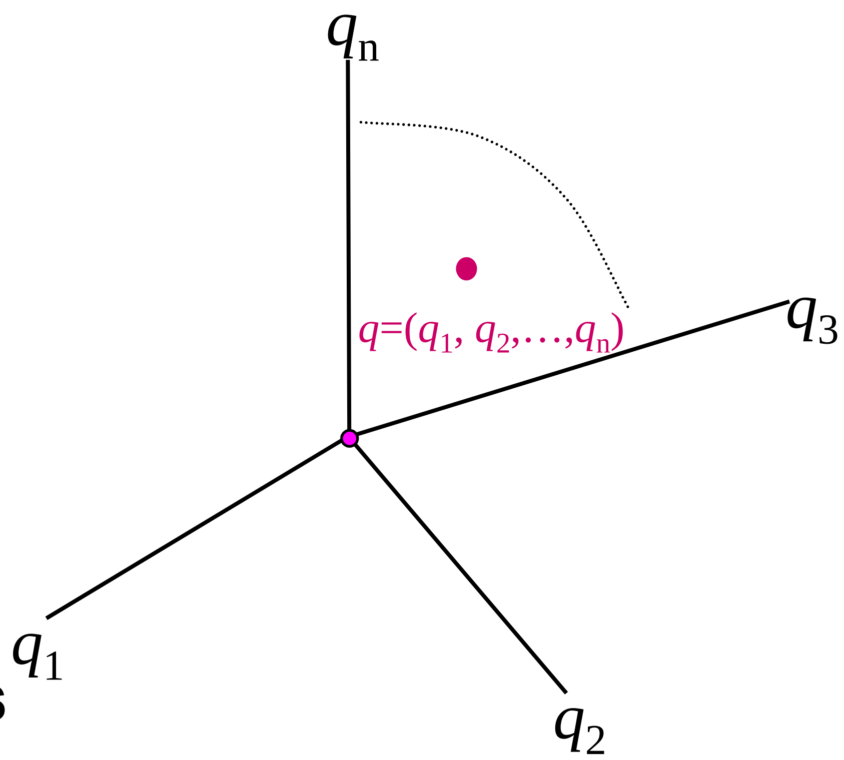


Configuration Space

- Definitions and examples
- Obstacles
- Paths
- Metrics

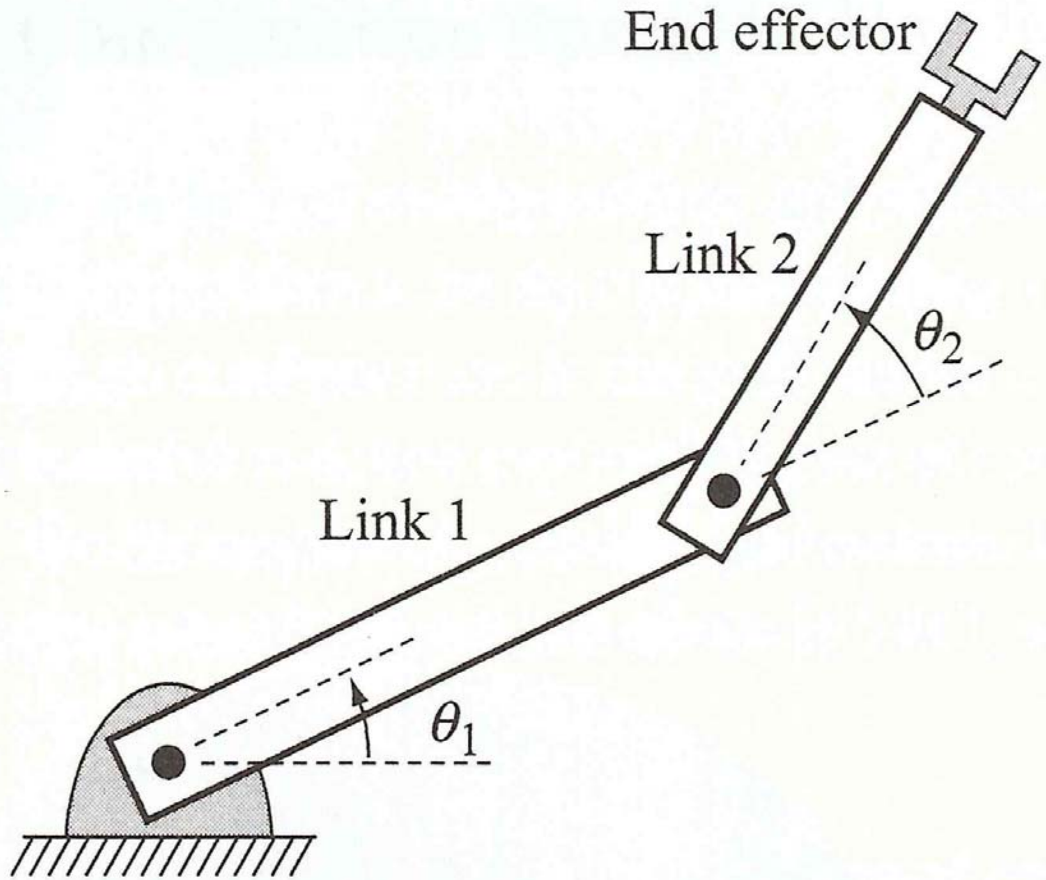
Configuration Space (C-space)

- **The configuration of an object is a complete specification of the position of every point on the object**
  - **Usually a configuration is expressed as a vector of position & orientation parameters:** $q = (q_1, q_2, \ldots, q_n)$

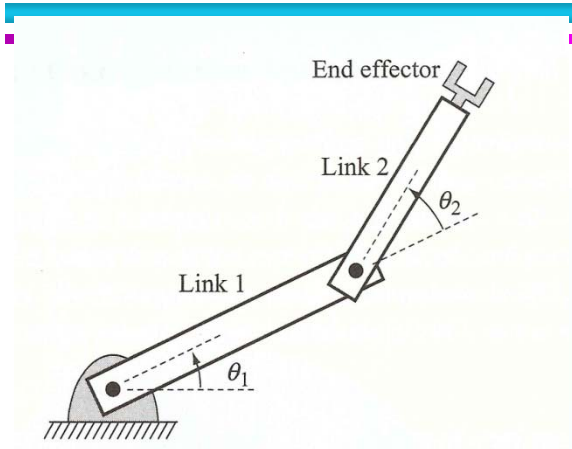


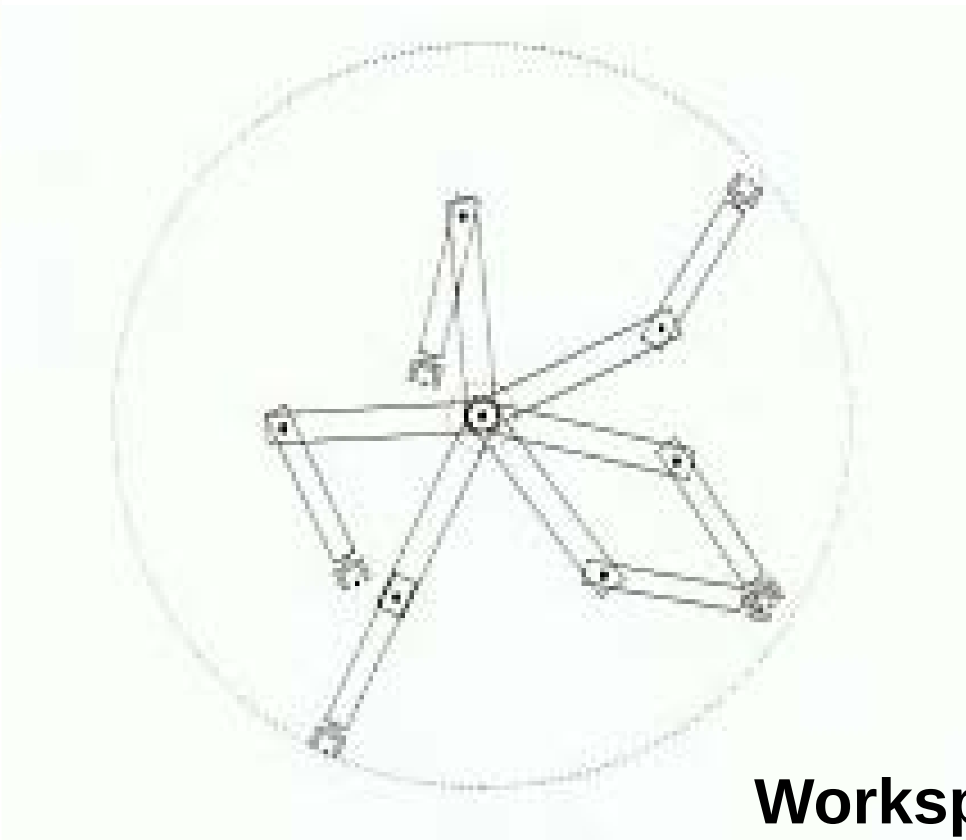
- **The configuration space** C **is the set of all possible configurations**
  - **A configuration is a point in** C

Examples of Configuration Spaces



Examples of Configuration Spaces

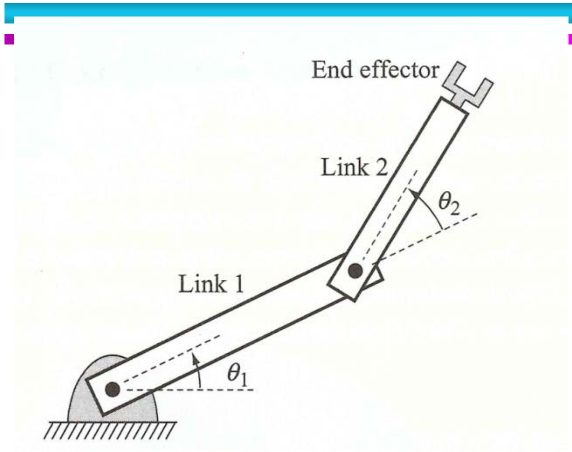




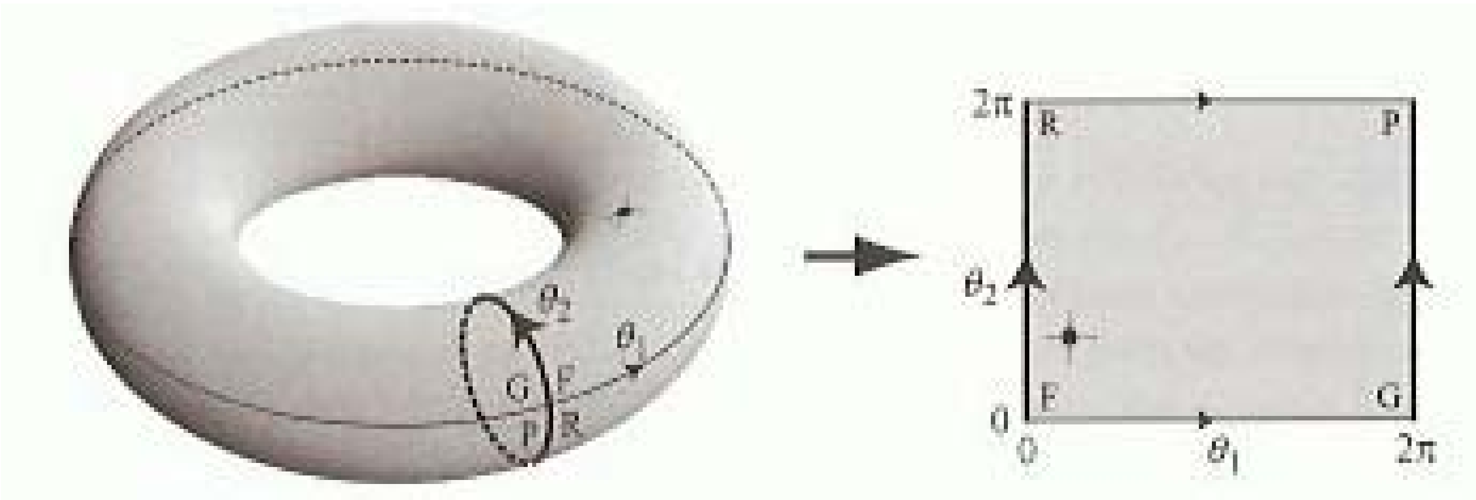
Workspace

This is not a valid C-space!

Examples of Configuration Spaces

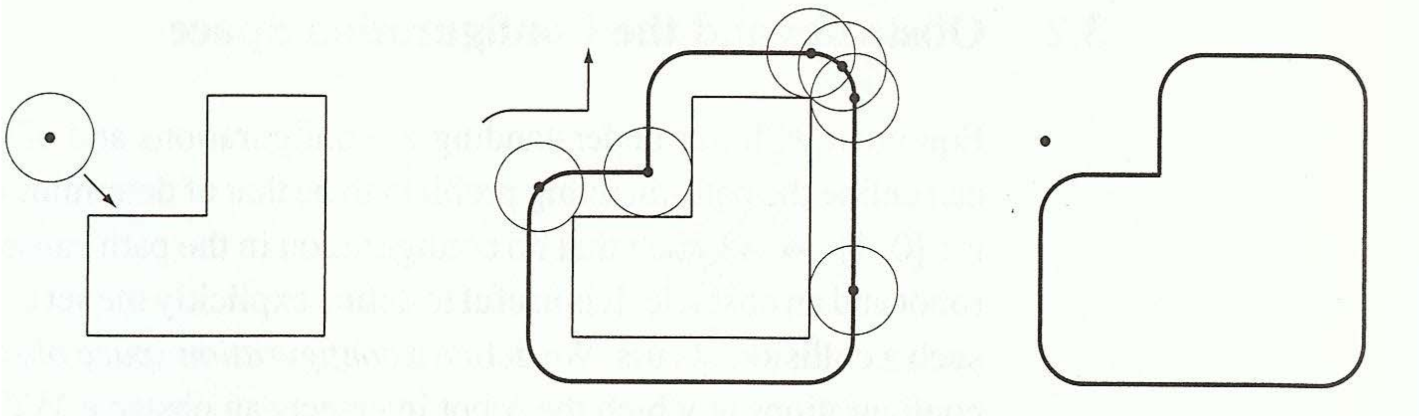


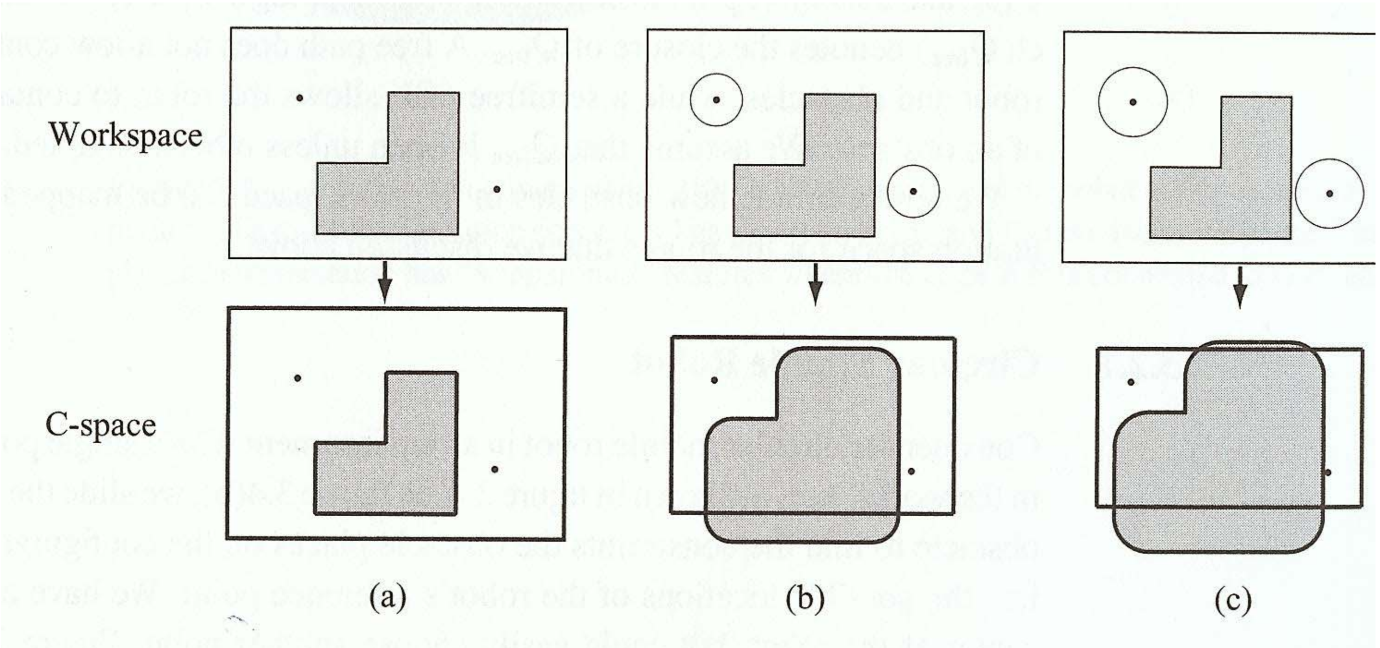
The topology of C is usually **not** that of a Cartesian space R^n.



$S^1 \times S^1 = T^2$

Examples of Circular Robot



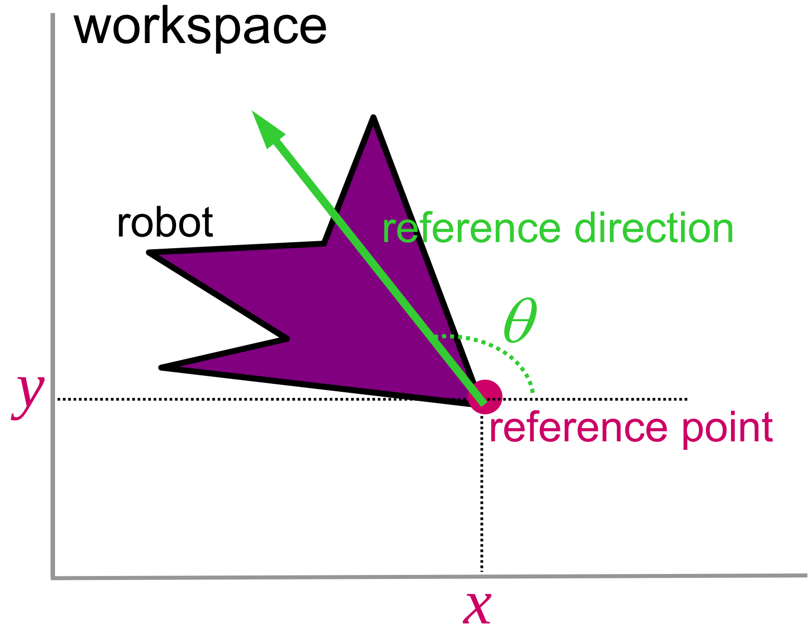


Dimension of Configuration Space

- The **dimension of the configuration space** is the **minimum** number of parameters needed to specify the configuration of the object completely

- It is also called the **number of degrees of freedom** (dofs) of a moving object

Example: Rigid Robot in 2-D Workspace



- **3-parameter specification:** $q = (x, y, \theta)$ **with** $\theta \in [0, 2\pi)$.
  - **3-D configuration space**

Example: Rigid Robot in 2-D workspace

- **4-parameter specification:** $q = (x, y, u, v)$ **with** $u^2+v^2 = 1$. **Note** $u = \cos\theta$ **and** $v = \sin\theta$

- **dim of configuration space = 3**
  - **Does the dimension of the configuration space (number of dofs) depend on the parametrization?**

Example: Rigid Robot in 2-D workspace

- **4-parameter specification:** $q = (x, y, u, v)$ **with** $u^2+v^2 = 1$. **Note** $u = \cos\theta$ **and** $v = \sin\theta$

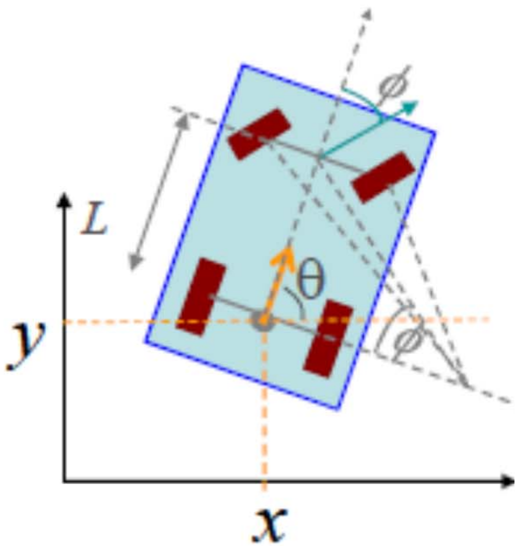
- **dim of configuration space =** **3**
  - **Does the dimension of the configuration space (number of dofs) depend on the parametrization?**
- **Topology: a 3-D cylinder** $C = \mathrm{R}^2$ **x** S^1

Holonomic and Non-Holonomic Contraints

- Holonomic constraints
  - g (q, t) = 0
- Non-holonomic constraints
  - g (q, q’, t) = 0
  - This is related to the kinematics of robots
  - To accommodate this, the C-space is extended to include the position and its velocity
- Dynamic constraints
  - Dynamic equations are represented as G(q, q’, q’’) = 0
  - These constraints are reduced to non-holonomic ones when we use the extended C-space

Example of Non-Holonomic Constraints

The path of the car is a curve tangent to its main rotation axis



$$dx \sin\theta - dy \cos\theta = 0$$

Example of Non-Holonomic Constraints

- **Point-mass robot with dynamics in a 2D plane**
  - **Its state is defined with its position and velocity (x, y, v_x, v_y)**
  - **To control the robot, we can apply forces in x- and y-directions**
  - **Then the equations of motions:**

$$x' = v_x \qquad v'_x = u_x / m$$

$$y' = v_y \qquad v'_y = u_y / m,$$

Where u_x and u_y are applied forces, and m is its mass

Computation of Dimension of C-Space

- Suppose that we have a rigid body that can translate and rotate in 2D workspace
  - Start with three points: A, B, C (6D space)

- We have the following (holonomic) constraints
  - Given A, we know the dist to B: d(A,B) = |A-B|
  - Given A and B, we have similar equations: d(A,C) = |A-C|, d(B,C) = |B-C|

- Each holonomic constraint reduces one dim.
  - Not for non-holonomic constraint

Example: Rigid Robot in 3-D Workspace

- **We can represent the positions and orientations of such robots with matrices (i.e., SO (3) and SE (3))**

SO (n) and SE (n)

- Special orthogonal group, SO(n), of n x n matrices R,

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

that satisfy:

$r_{1i}^2 + r_{2i}^2 + r_{3i}^2 = 1$ for all i ,
$r_{1i} r_{1j} + r_{2i} r_{2j} + r_{3i} r_{3j} = 0$ for all $i \neq j$,
$\det(R) = +1$

Refer to the 3D Transformation at the undergraduate computer graphics.

- Given the orientation matrix R of SO (n) and the position vector p, special Euclidean group, SE (n), is defined as:

$$\begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

Example: Rigid Robot in 3-D Workspace

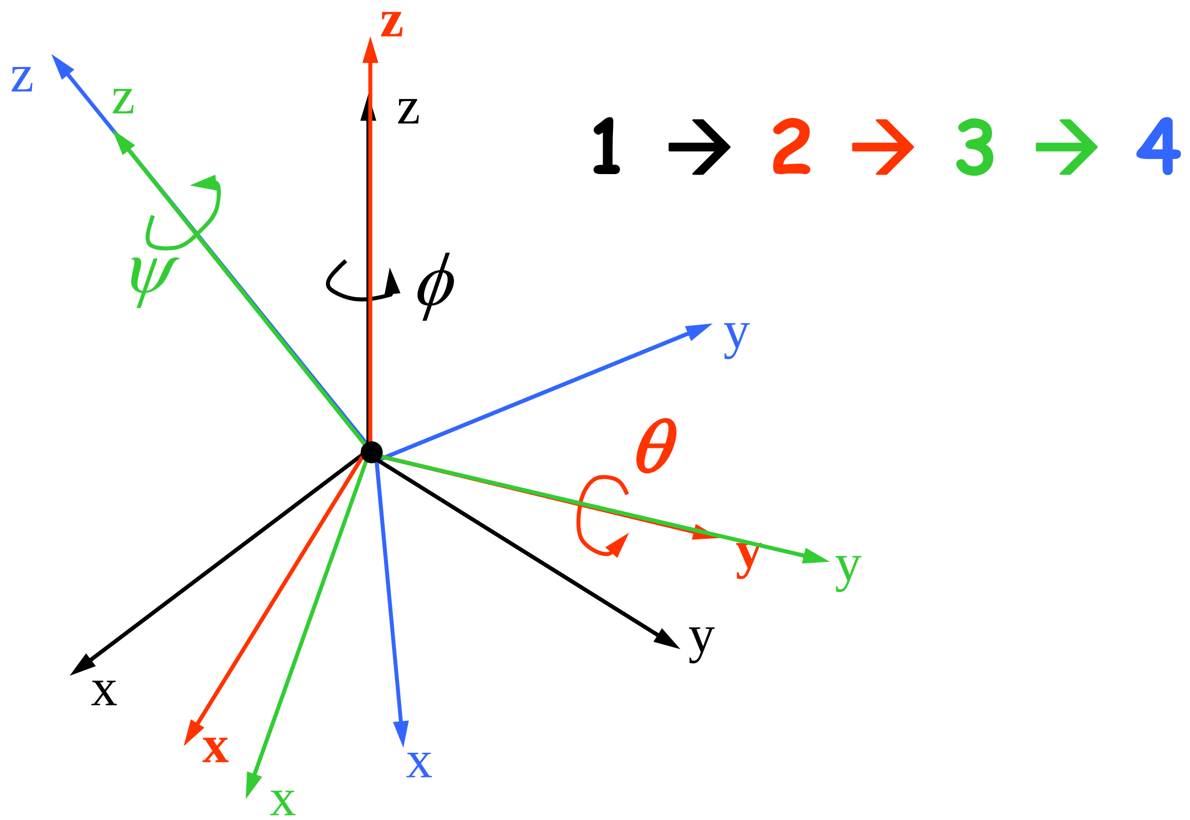
- q = (position, orientation) = ($x, y, z,$ **???**)

- **Parametrization of orientations by matrix:**
  $q = (r_{11}, r_{12}, \ldots, r_{33}, r_{33})$ **where** $r_{11}, r_{12}, \ldots, r_{33}$ **are the elements of rotation matrix**

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \in SO(3)$$

Example: Rigid Robot in 3-D Workspace

- **Parametrization of orientations by Euler angles:** (ϕ, θ, ψ)



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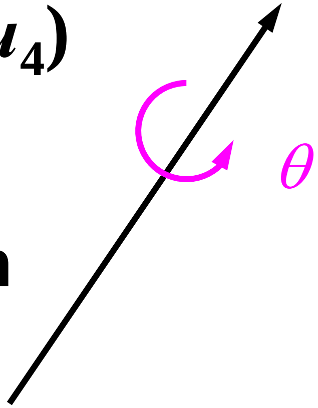
Example: Rigid Robot in 3-D Workspace

- **Parametrization of orientations by unit quaternion:** $u = (u_1, u_2, u_3, u_4)$ **with** $u_1^2 + u_2^2 + u_3^2 + u_4^2 = 1$.
  - **Note** $(u_1, u_2, u_3, u_4) = (\cos\theta/2, n_x\sin\theta/2, n_y\sin\theta/2, n_z\sin\theta/2)$ **with** $n_x^2 + n_y^2 + n_z^2 = 1$
  - **Compare with representation of orientation in 2-D:** $(u_1, u_2) = (\cos\theta, \sin\theta)$

$\mathbf{n} = (n_x, n_y, n_z)$



Example: Rigid Robot in 3-D Workspace

- **Advantage of unit quaternion representation**
  - **Compact**
  - **No singularity**
  - **Naturally reflect the topology of the space of orientations**

- **Number of dofs = 6**
- **Topology: $R^3 \times SO(3)$**

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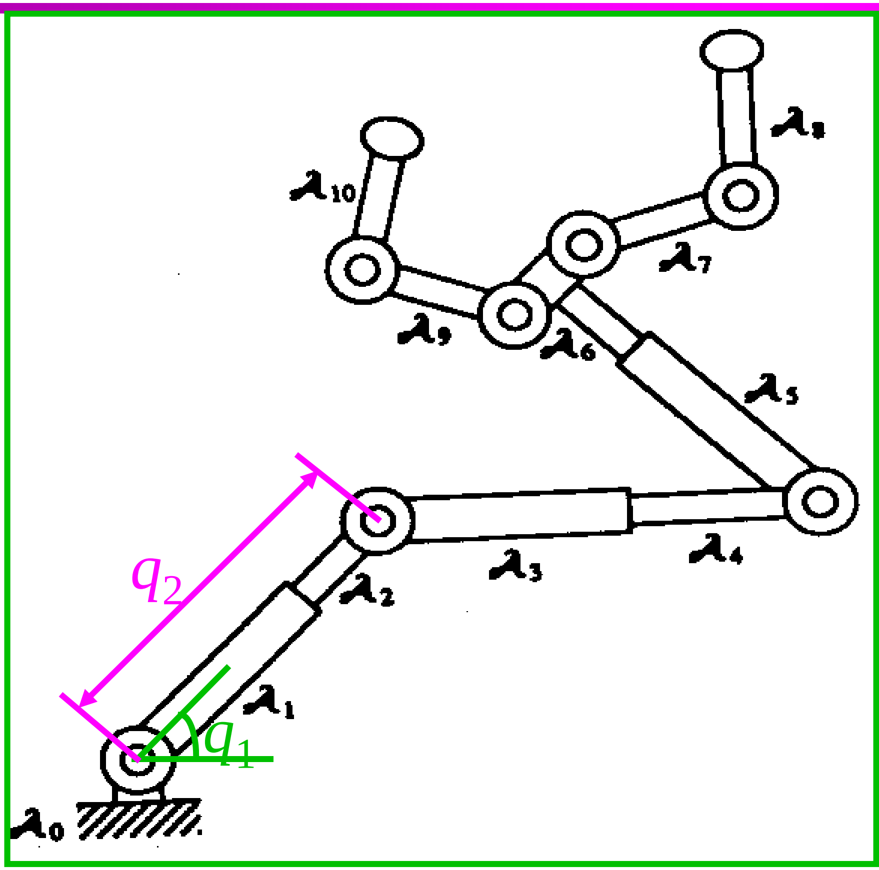
Example: Articulated Robot



An articulated object is a set of rigid bodies connected at the joints.

- $q = (q_1, q_2, \ldots, q_{2n})$
- **Number of dofs** = $2n$
- **What is the topology?**

Class Objectives were:

- Configuration space
  - Definitions and examples
  - Obstacles
  - Paths
  - Metrics

Next Time….

- **Configuration space**
  - **Definitions and examples**
  - **Obstacles**
  - **Paths**
  - **Metrics**

Homework for Every Class

- **Go over the next lecture slides**
- **Come up with one question on what we have discussed today and submit at the end of the class**
  - **1 for typical questions**
  - **2 for questions with thoughts or that surprised me**

- **Write a question at least 10 times**
  - **Do that out of 2 classes**
  - **Online: http://bit.ly/1evlQ5D**