Configuration Space II

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Class Objectives

- **Configuration space**
  - **Definitions and examples**
  - **Obstacles**
  - **Paths**
  - **Metrics**

Configuration Space

- Definitions and examples
- Obstacles
- Paths
- Metrics

Obstacles in the Configuration Space

- **A configuration q is collision-free, or free, if a moving object placed at q does not intersect any obstacles in the workspace**

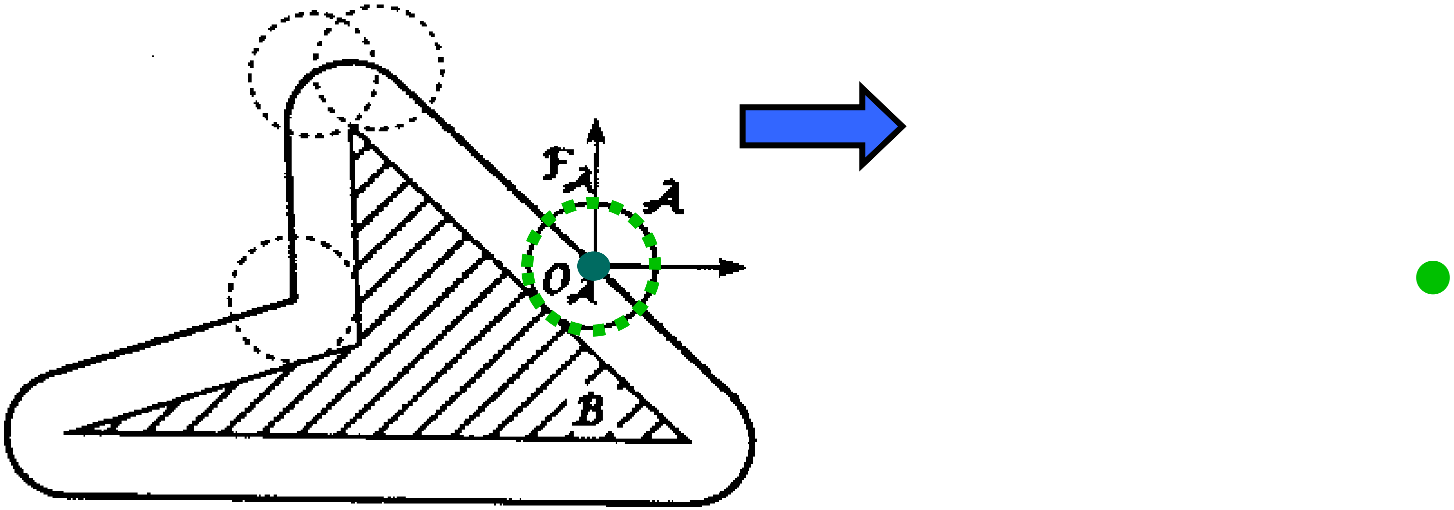
- **The free space F is the set of free configurations**

- **A configuration space obstacle (C-obstacle) is the set of configurations where the moving object collides with workspace obstacles**

Disc in 2-D Workspace

workspace

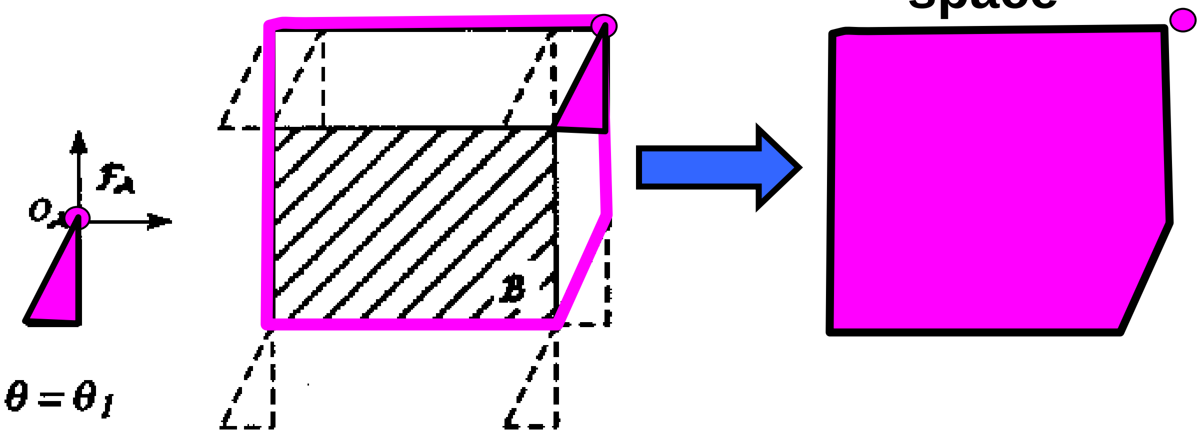
configuration space



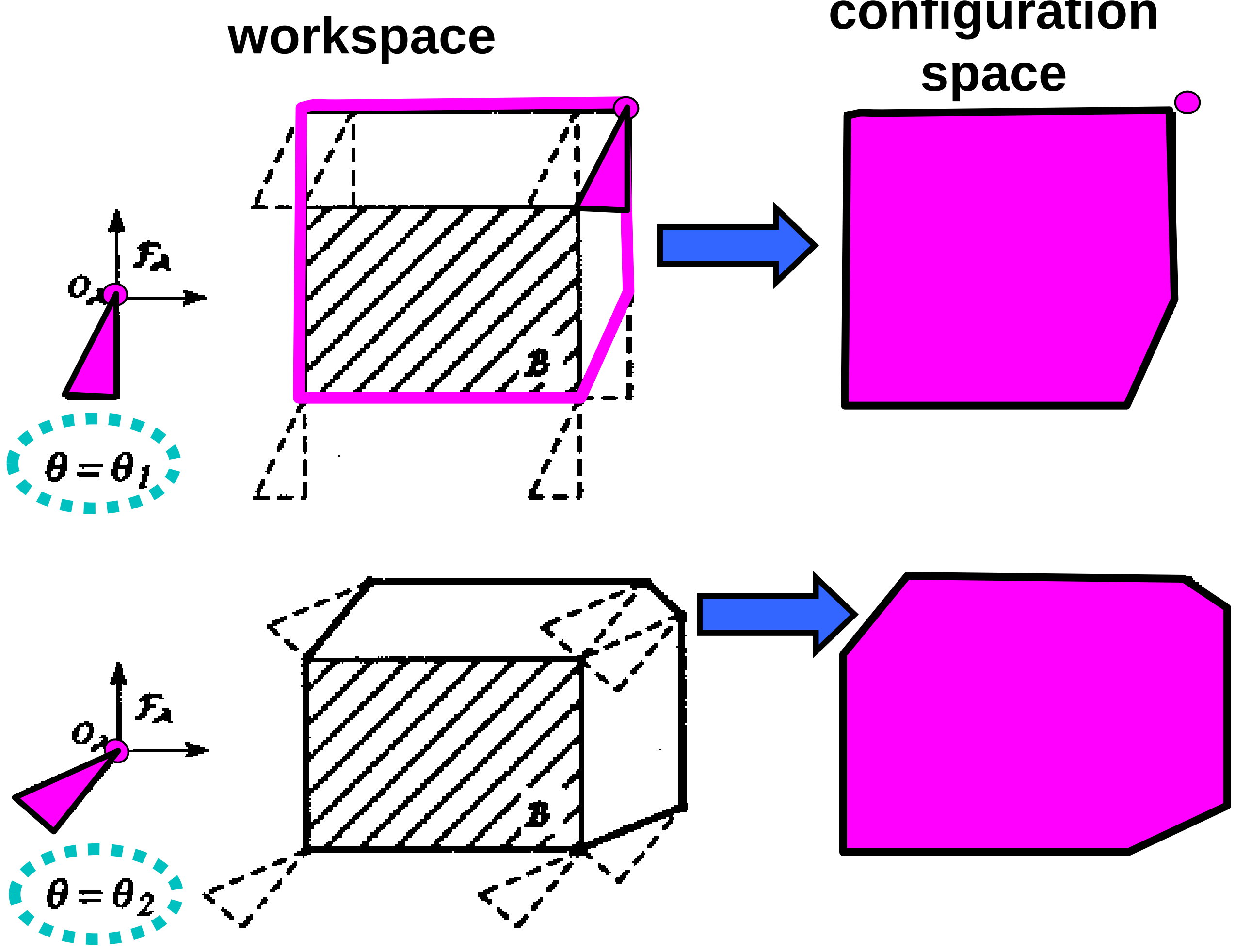
Polygonal Robot Translating in 2-D Workspace

workspace

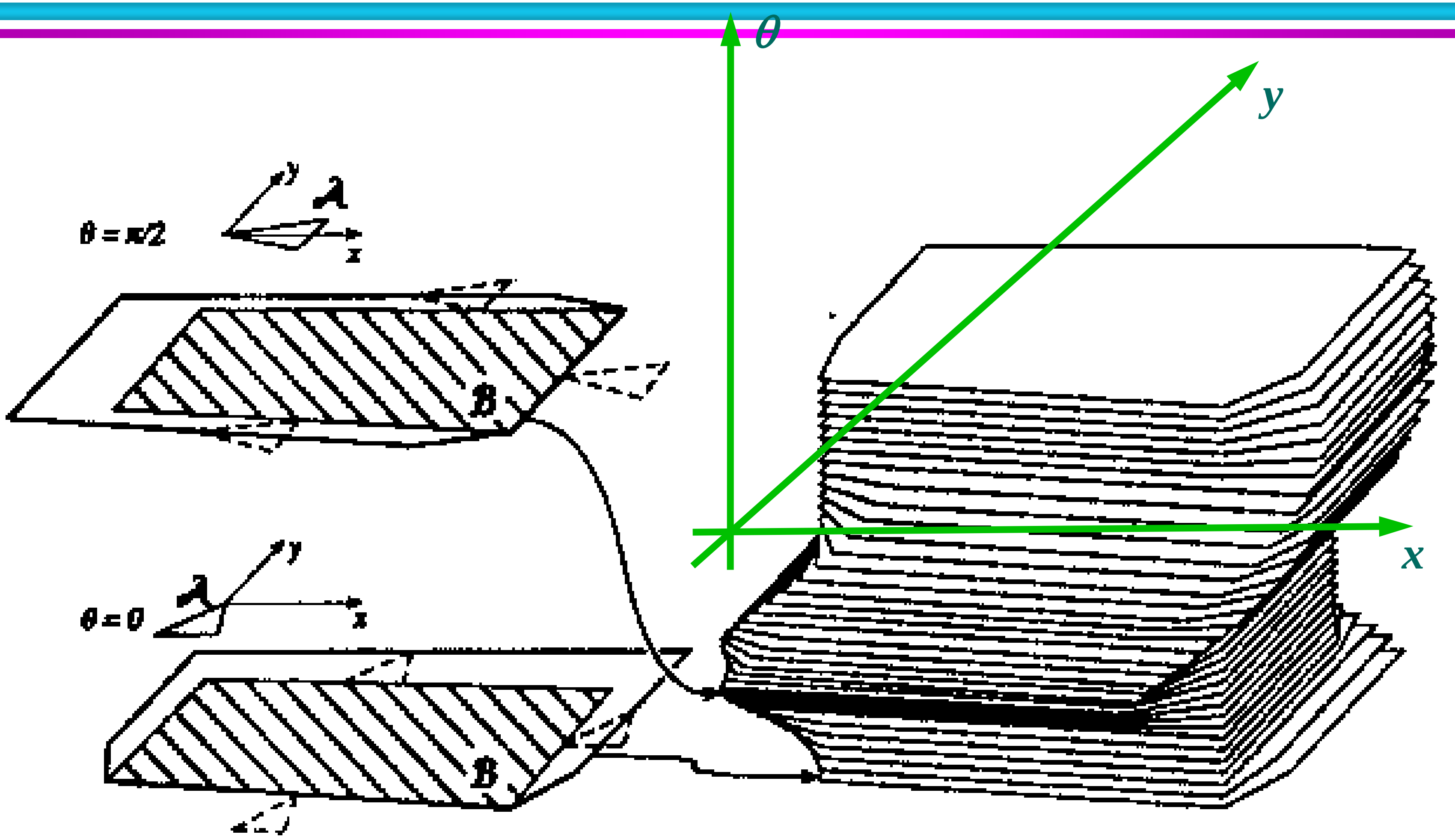
configuration space



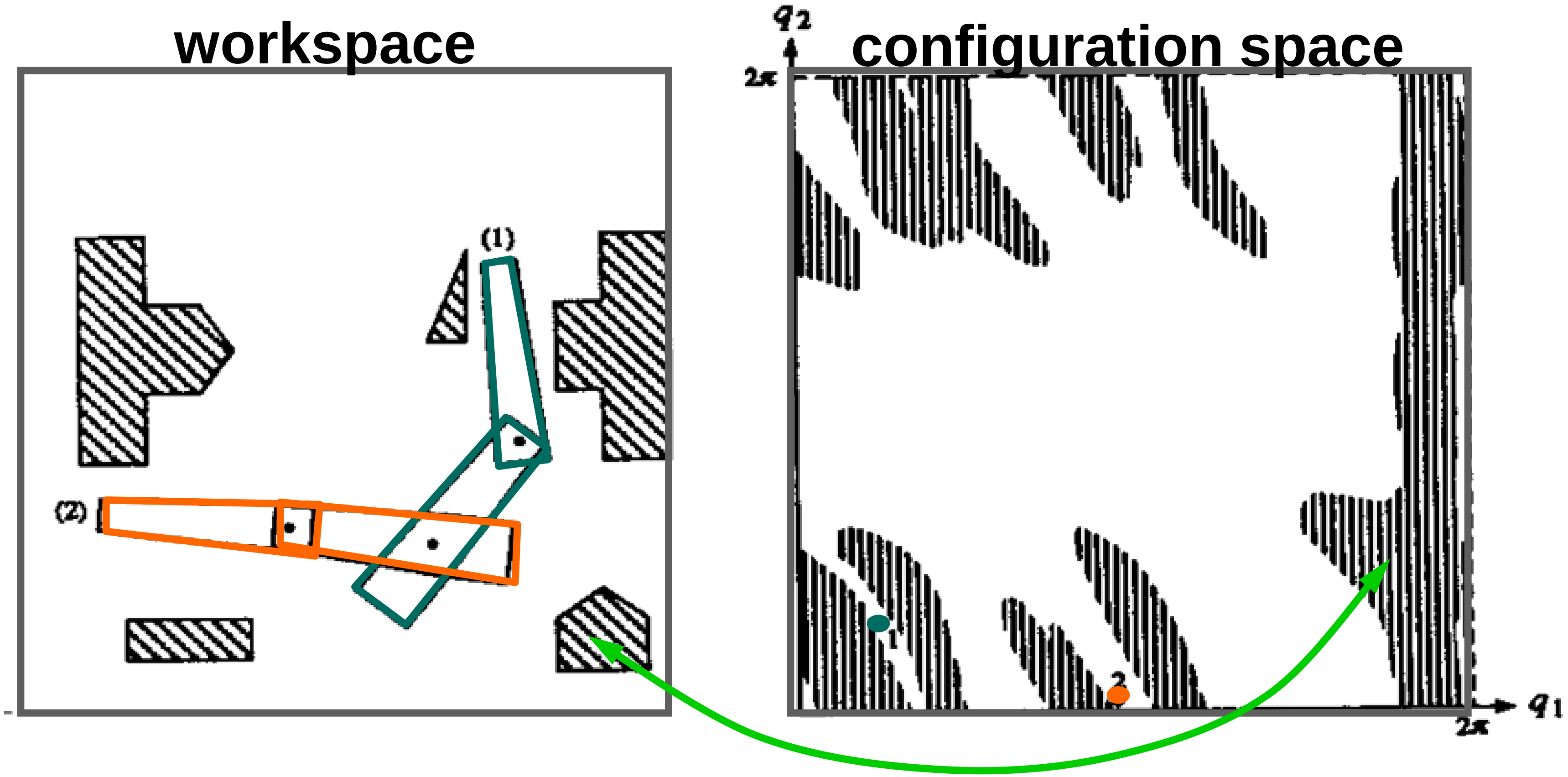
Polygonal Robot Translating & Rotating in 2-D Workspace



Polygonal Robot Translating & Rotating in 2-D Workspace



Articulated Robot in 2-D Workspace



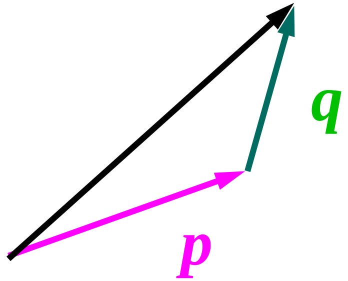
C-Obstacle Construction

- **Input:**
  - **Polygonal moving object translating in 2-D workspace**
  - **Polygonal obstacles**
- **Output: configuration space obstacles represented as polygons**

Minkowski Sum

- **The Minkowski sum of two sets P and Q, denoted by $P \oplus Q$, is defined as**

$$P \oplus Q = \{\, p+q \mid p \in P,\ q \in Q \,\}$$



- **Similarly, the Minkowski difference is defined as**

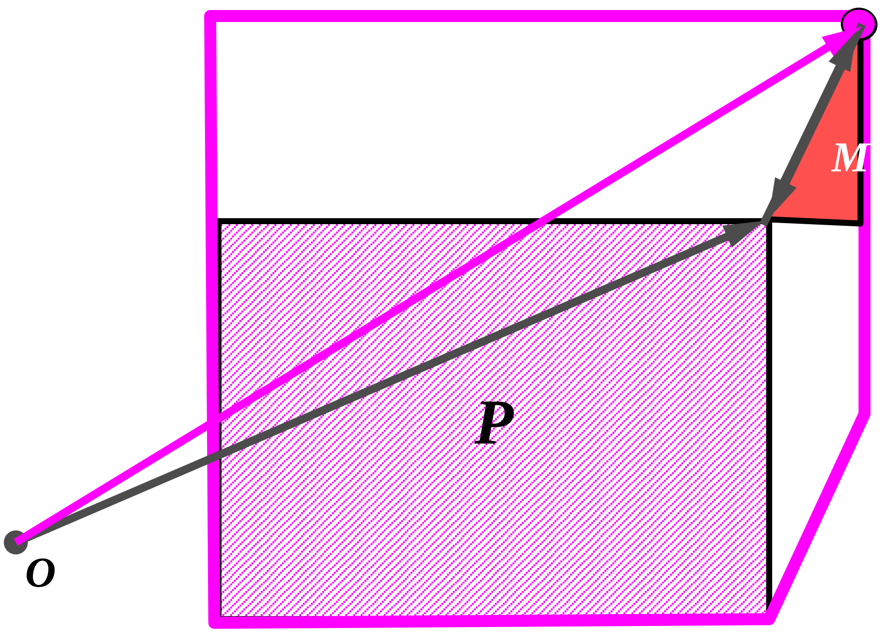
$$P \ominus Q = \{\, p-q \mid p \in P,\ q \in Q \,\}$$
$$= P \oplus -Q$$

Minkowski Sum of Convex polygons

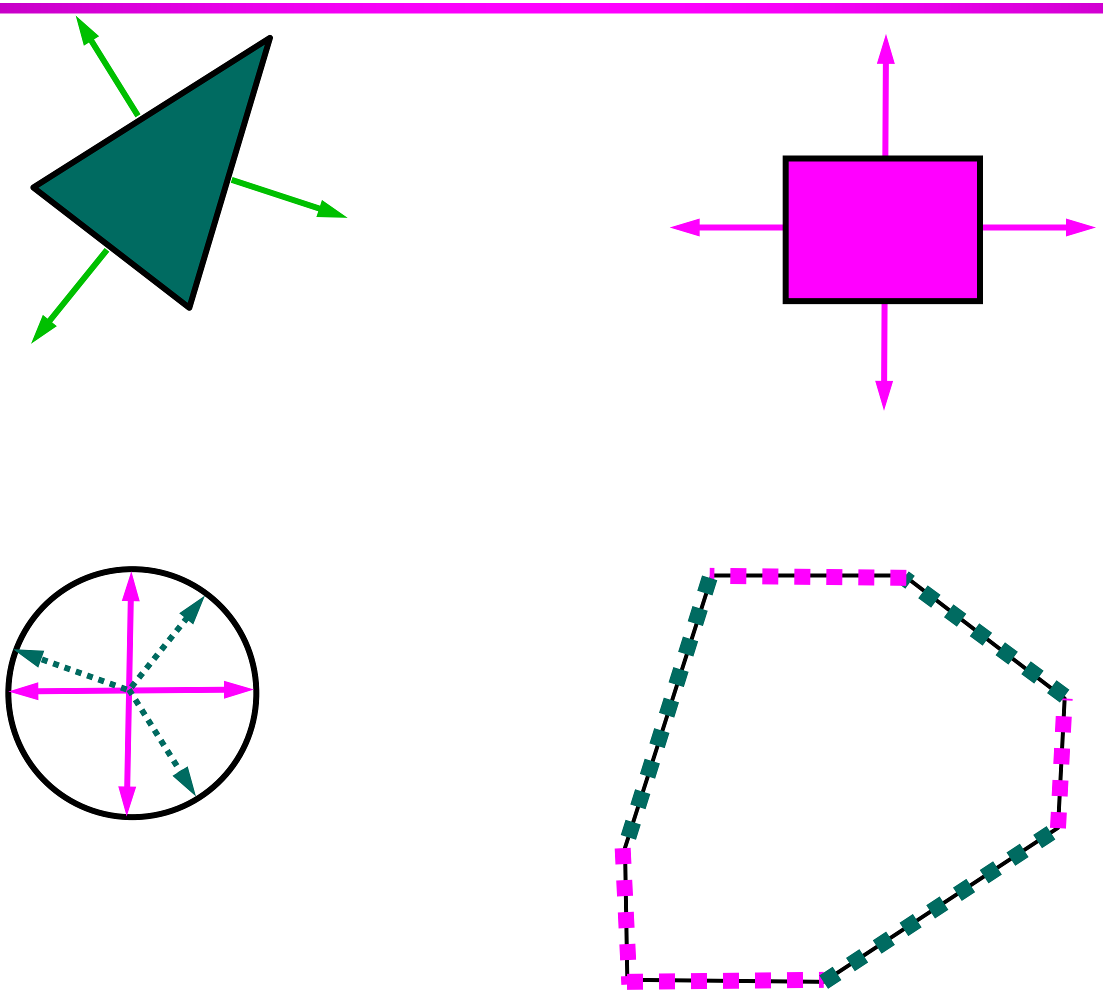
- The Minkowski sum of two convex polygons P and Q of m and n vertices respectively is a convex polygon $P \oplus Q$ of $m + n$ vertices.
  - The vertices of $P \oplus Q$ are the "sums" of vertices of P and Q.

Observation

- If P is an obstacle in the workspace and M is a moving object. Then the C-space obstacle corresponding to P is $P \ominus M$.



Computing C-obstacles



Computational efficiency

- Running time $O(n+m)$
- Space $O(n+m)$
- Non-convex obstacles
  - Decompose into convex polygons (*e.g.,* triangles or trapezoids), compute the Minkowski sums, and take the union
  - Complexity of Minkowksi sum $O(n^2m^2)$

- 3-D workspace

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Configuration space

- **Definitions and examples**
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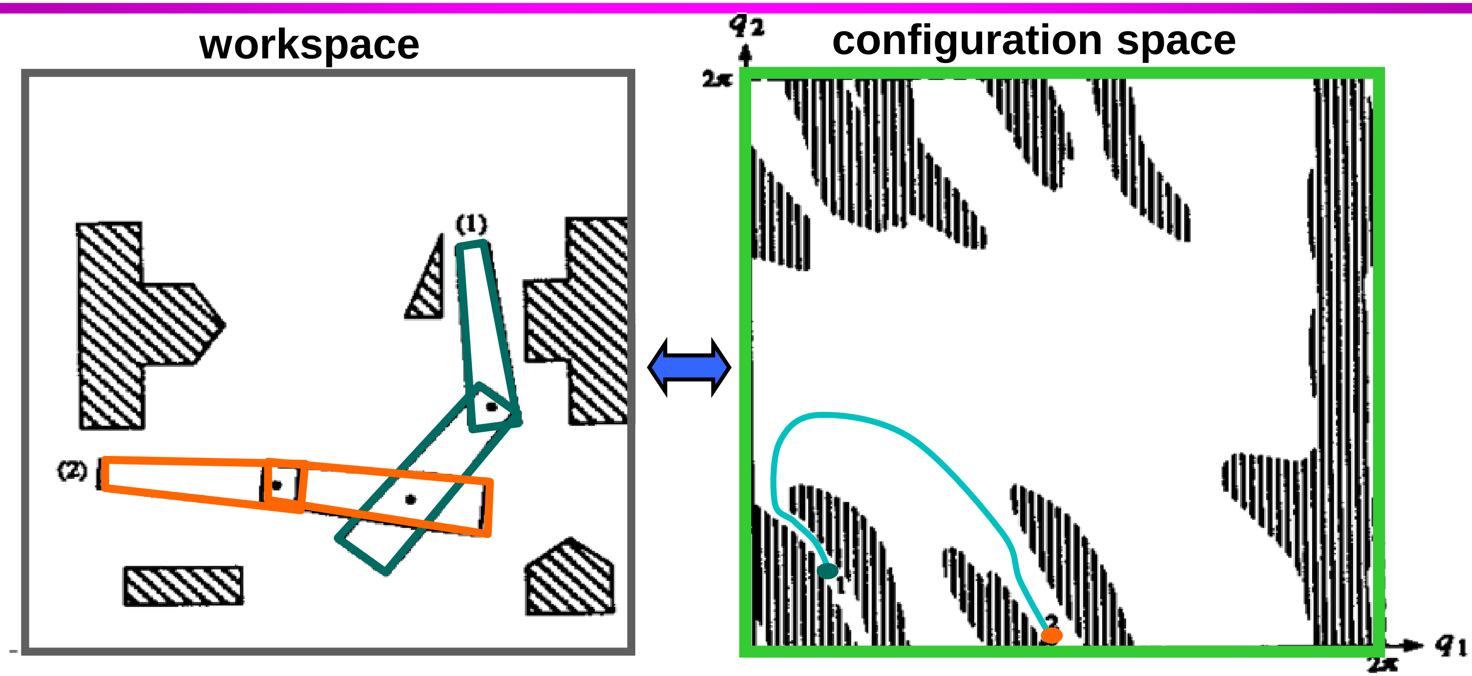
Paths in the configuration space



- A **path** in C is a continuous curve connecting two configurations q and q':

$$\tau : s \in [0,1] \rightarrow \tau(s) \in C$$

such that $\tau(0) = q$ **and** $\tau(1) = q'$**.**

Constraints on paths

- A **trajectory** is a path parameterized by time:

$$\tau : t \in [0, T] \rightarrow \tau(t) \in C$$

- Constraints
  - Finite length
  - Bounded curvature
  - Smoothness
  - Minimum length
  - Minimum time
  - Minimum energy
  - ...

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Free Space Topology

- **A free path lies entirely in the free space F.**
- **The moving object and the obstacles are modeled as closed subsets, meaning that they contain their boundaries.**
- **One can show that the C-obstacles are closed subsets of the configuration space C as well.**
- **Consequently, the free space F is an open subset of C.**

Semi-Free Space

- **A configuration q is semi-free if the moving object placed q touches the boundary, but not the interior of obstacles.**
  - **Free, or**
  - **In contact**
- **The semi-free space is a closed subset of C.**

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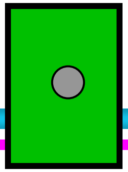
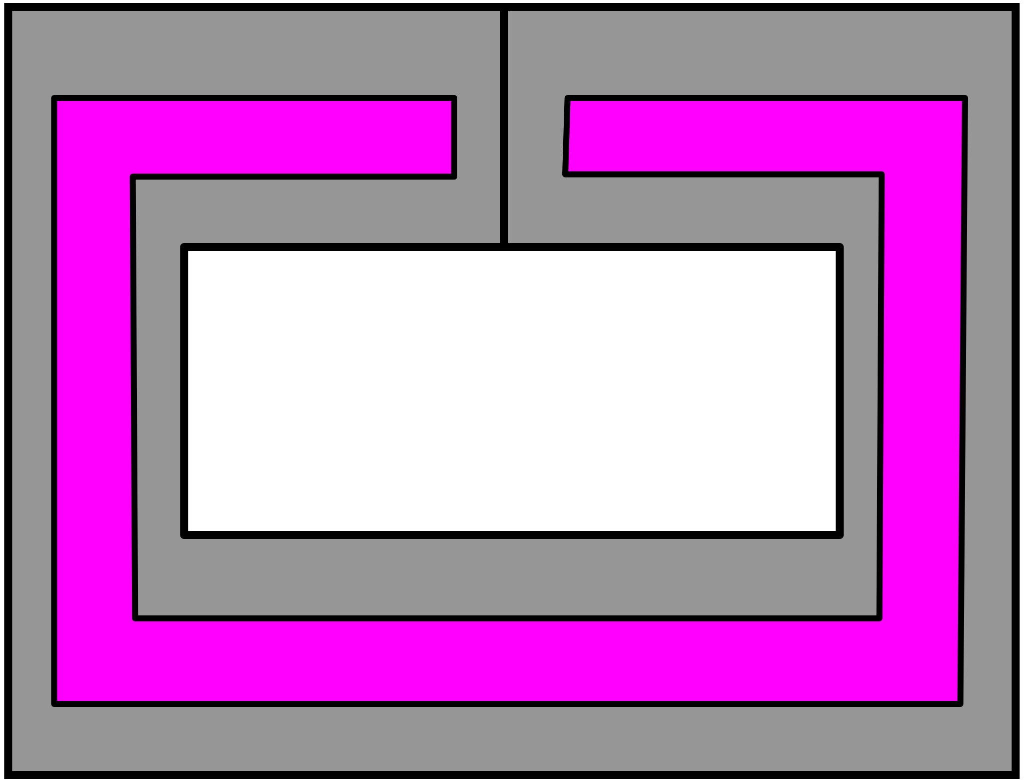
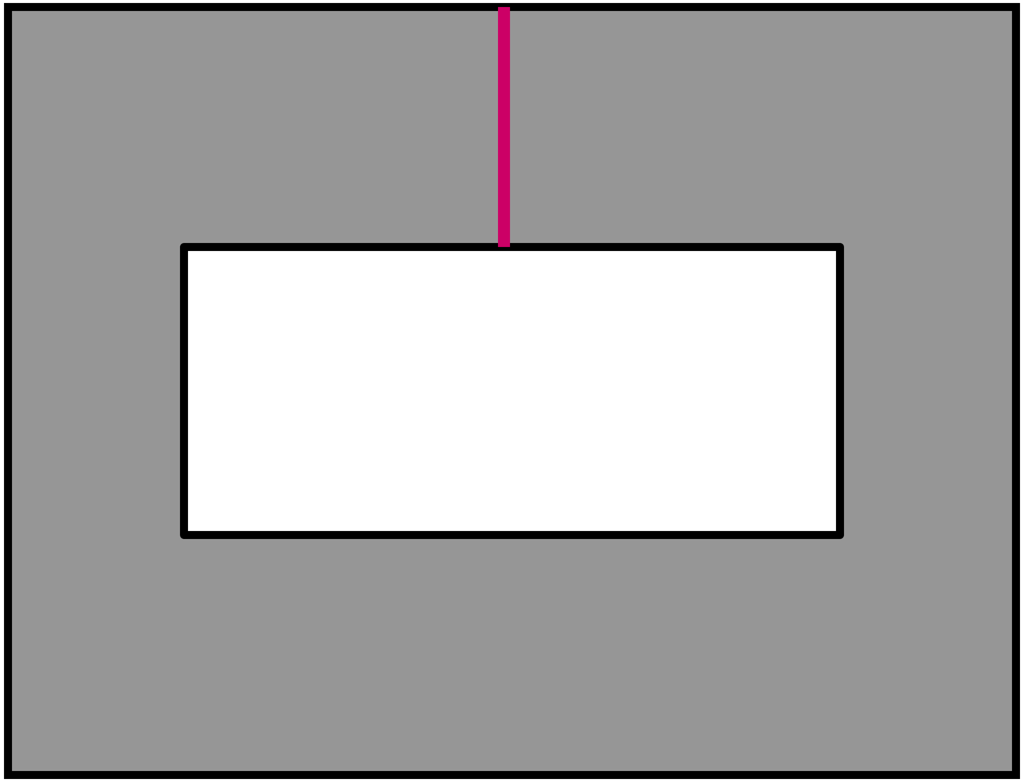
Example





Example

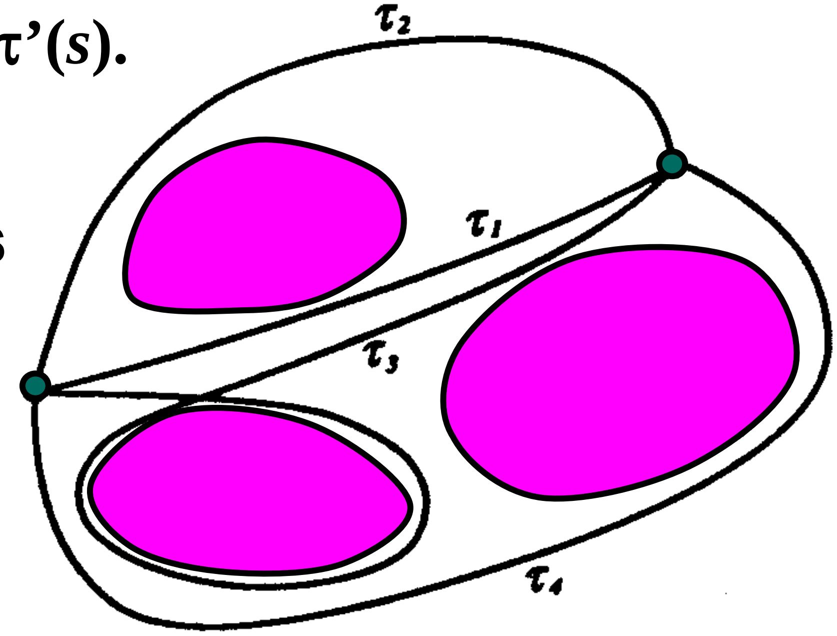


Homotopic Paths

- **Two paths τ and τ' (that map from U to V) with the same endpoints are homotopic if one can be continuously deformed into the other:**

$$h : U \times [0,1] \rightarrow V$$

with $h(s,0) = \tau(s)$ and $h(s,1) = \tau'(s)$.



- **A homotopic class of paths contains all paths that are homotopic to one another.**

Connectedness of C-Space

- C is **connected** if every two configurations can be connected by a path.
- C is **simply-connected** if any two paths connecting the same endpoints are homotopic.
  Examples: R^2 or R^3
- Otherwise C is multiply-connected.

Configuration space

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Metric in Configuration Space

- A **metric** or **distance** function d in a configuration space C is a function

$$d : (q, q') \in C^2 \rightarrow d(q, q') \geq 0$$

such that
  - $d(q, q') = 0$ if and only if $q = q'$,
  - $d(q, q') = d(q', q)$,
  - $d(q, q') \leq d(q, q'') + d(q'', q')$.

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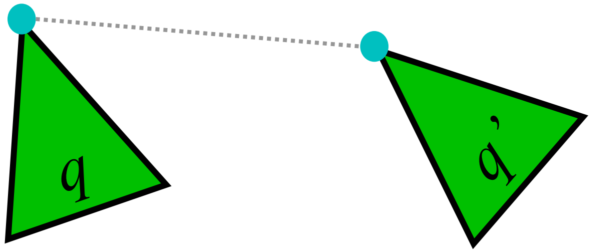
Example

- **Robot** A **and a point** x **on** A
- $x(q)$**: position of** x **in the workspace when** A **is at configuration** q
- **A distance** d **in** C **is defined by**

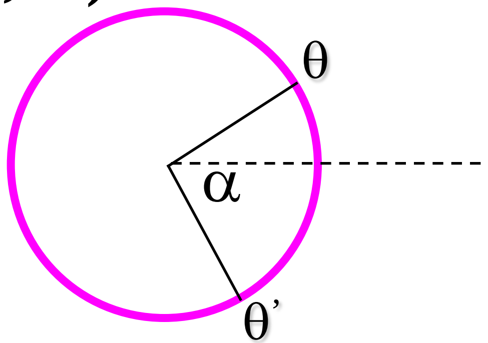
$$d(q, q') = \max_{x \in A} \| x(q) - x(q') \|$$

, where $||x - y||$ **denotes the Euclidean distance between points** x **and** y **in the workspace.**



Examples in R^2 x S^1

- **Consider R^2 x S^1**
  - $q = (x, y, \theta)$, $q' = (x', y', \theta')$ **with** $\theta, \theta' \in [0,2\pi)$
  - $\alpha = \min \{ \, |\theta - \theta'| \, , 2\pi - |\theta - \theta'| \, \}$



- $d(q, q') = \mathrm{sqrt}((x-x')^2 + (y-y')^2 + \alpha^2))$

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Next Time….

- **Collision detection and distance computation**