<Special Topic in Rendering>

Path Guiding

CS482 Interactive Computer Graphics

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Review

Rendering Equation

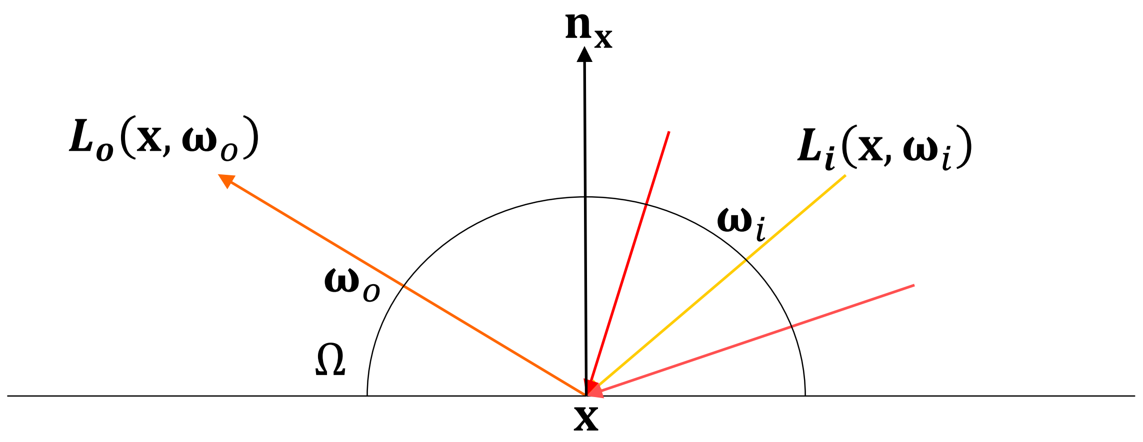
Monte Carlo Path Tracing

Rendering Equation

- Rendering equation
  [Immel et al. 1986; Kajiya 1986]

L: a radiance
$\mathbf{x}$: a point
$\boldsymbol{\omega}$: a direction
Ω: the hemisphere
$\mathbf{n}_\mathbf{x}$: the normal vector at $\mathbf{x}$
f_s: the BSDF (material)
i: inward
o: outward
e: self-emitting

- $\underbrace{L_o(\mathbf{x}, \boldsymbol{\omega}_o)}_{\text{Outgoing radiance}} = \underbrace{L_e(\mathbf{x}, \boldsymbol{\omega}_o)}_{\text{Self-emitting radiance}} + \int_\Omega \underbrace{L_i(\mathbf{x}, \boldsymbol{\omega}_i)}_{\text{Inward radiance}} \underbrace{f_s(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o)}_{\text{BSDF = Material}} (\boldsymbol{\omega}_i \cdot \mathbf{n}_\mathbf{x}) d\boldsymbol{\omega}_i$

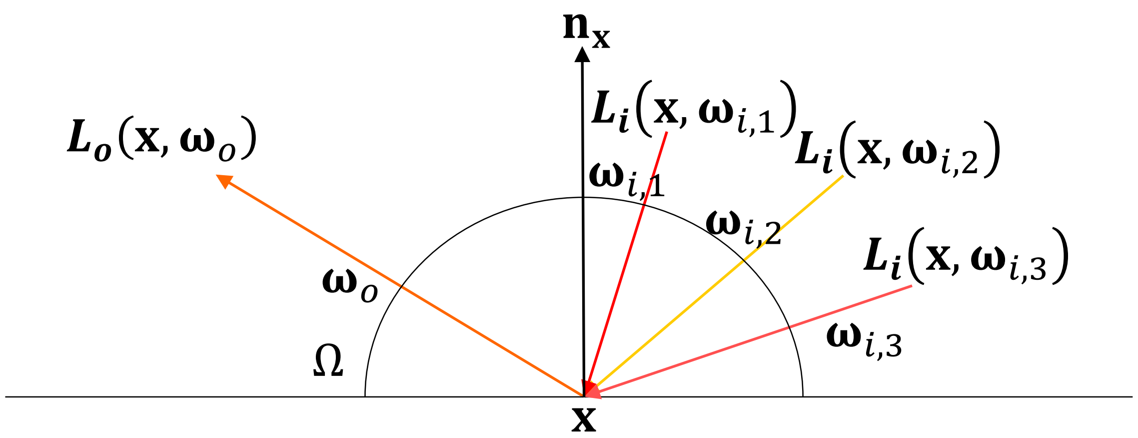


Monte Carlo Ray Tracing

L: a radiance
$\mathbf{x}$: a point
$\boldsymbol{\omega}$: a direction
Ω: the hemisphere
$\mathbf{n_x}$: the normal vector at $\mathbf{x}$
f_s: the BSDF (material)
i: inward
o: outward
e: self-emitting

- MC integration of rendering equation.

- $\langle L_o(\mathbf{x}, \boldsymbol{\omega}_o)\rangle = L_e(\mathbf{x}, \boldsymbol{\omega}_o) + \frac{1}{N}\sum_{k=1}^{N} \frac{\langle L_i(\mathbf{x},\boldsymbol{\omega}_{i,k})\rangle f_s(\mathbf{x},\boldsymbol{\omega}_{i,k},\boldsymbol{\omega}_o)(\boldsymbol{\omega}_{i,k}\cdot\mathbf{n_x})}{p(\boldsymbol{\omega}_{i,k})}$

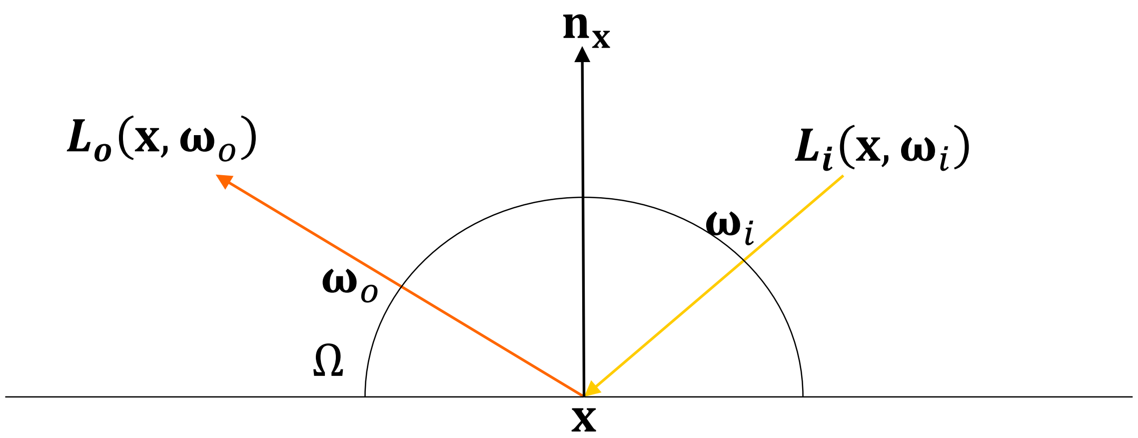


Monte Carlo Path Tracing

L: a radiance
$\mathbf{x}$: a point
$\boldsymbol{\omega}$: a direction
Ω: the hemisphere
$\mathbf{n}_\mathbf{x}$: the normal vector at $\mathbf{x}$
f_s: the BSDF (material)
i: inward
o: outward
e: self-emitting

- MC integration of rendering equation.

- Set $N = 1$ for intermediate bounces
- $\langle L_o(\mathbf{x}, \boldsymbol{\omega}_o)\rangle = L_e(\mathbf{x}, \boldsymbol{\omega}_o) + \frac{\langle L_i(\mathbf{x},\boldsymbol{\omega}_i)\rangle f_s(\mathbf{x},\boldsymbol{\omega}_i,\boldsymbol{\omega}_o)(\boldsymbol{\omega}_i \cdot \mathbf{n}_\mathbf{x})}{p(\boldsymbol{\omega}_i)}$



Path Guiding

Variance in Path Tracing

- $\langle L_o(\mathbf{x}, \boldsymbol{\omega}_o)\rangle = L_e(\mathbf{x}, \boldsymbol{\omega}_o) + \frac{1}{N}\sum_{k=1}^{N} \frac{\langle L_i(\mathbf{x},\boldsymbol{\omega}_{i,k})\rangle f_s(\mathbf{x},\boldsymbol{\omega}_{i,k},\boldsymbol{\omega}_o)(\boldsymbol{\omega}_{i,k}\cdot\mathbf{n}_\mathbf{x})}{p(\boldsymbol{\omega}_{i,k})}$
- $Var[\langle L_o(\mathbf{x}, \boldsymbol{\omega}_o)\rangle] = Var\left[\frac{1}{N}\sum_{k=1}^{N} \frac{\langle L_i(\mathbf{x},\boldsymbol{\omega}_{i,k})\rangle f_s(\mathbf{x},\boldsymbol{\omega}_{i,k},\boldsymbol{\omega}_o)(\boldsymbol{\omega}_{i,k}\cdot\mathbf{n}_\mathbf{x})}{p(\boldsymbol{\omega}_{i,k})}\right]$

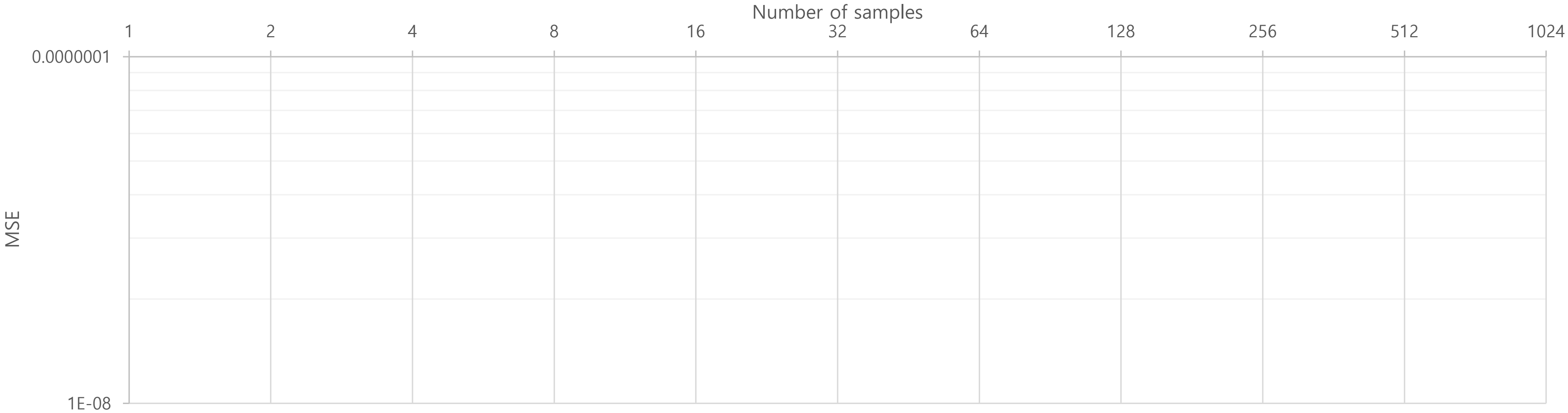
$$= \frac{1}{N} Var\left[\frac{\langle L_i(\mathbf{x},\boldsymbol{\omega}_i)\rangle f_s(\mathbf{x},\boldsymbol{\omega}_i,\boldsymbol{\omega}_o)(\boldsymbol{\omega}_i\cdot\mathbf{n}_\mathbf{x})}{p(\boldsymbol{\omega}_i)}\right]$$

- If $p \propto L_i f_s \cos\theta_i$, $Var[\langle L_o(\mathbf{x}, \boldsymbol{\omega}_o)\rangle] = 0$
  - Shoot more rays to the direction with intense light

- Path guiding: estimation for incident radiance

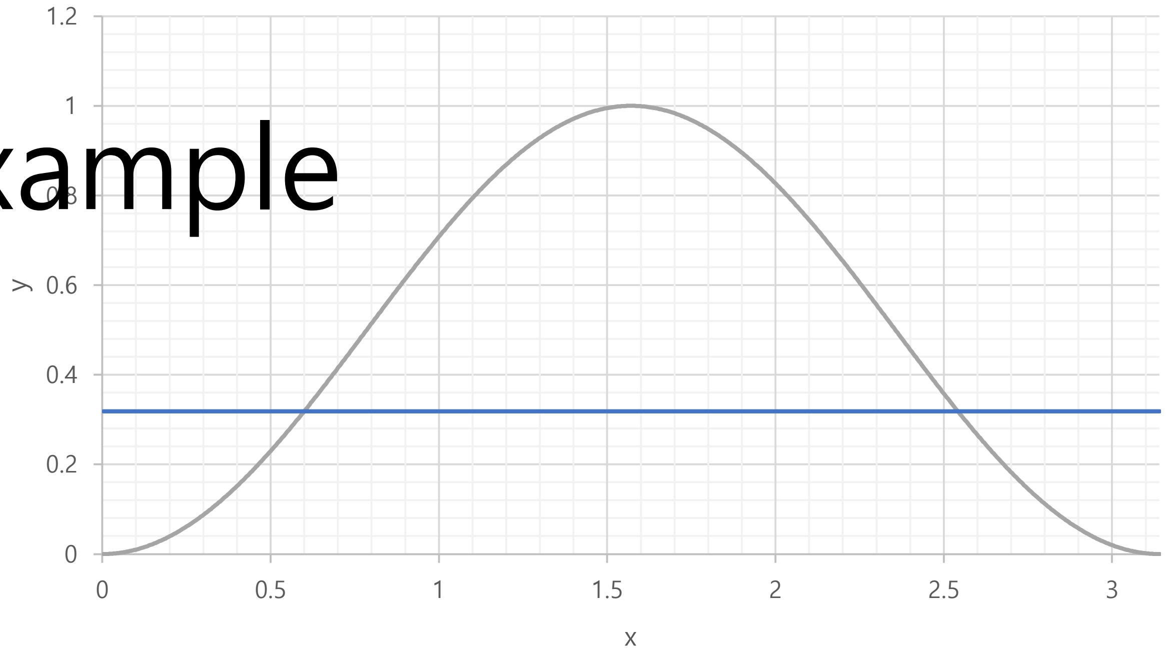
Path Guiding – 1D example

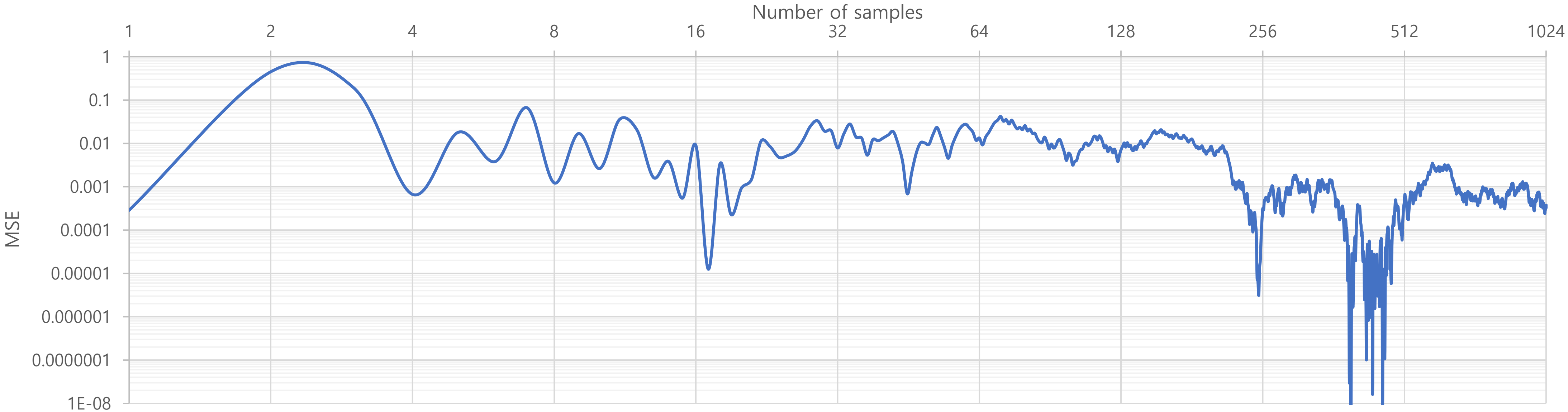
- MC integration for $\int_0^\pi \sin^2 x \, dx$
  - $\frac{1}{N}\sum_{n=1}^{N}\frac{\sin^2 x_n}{p(x_n)}$



Path Guiding – 1D example

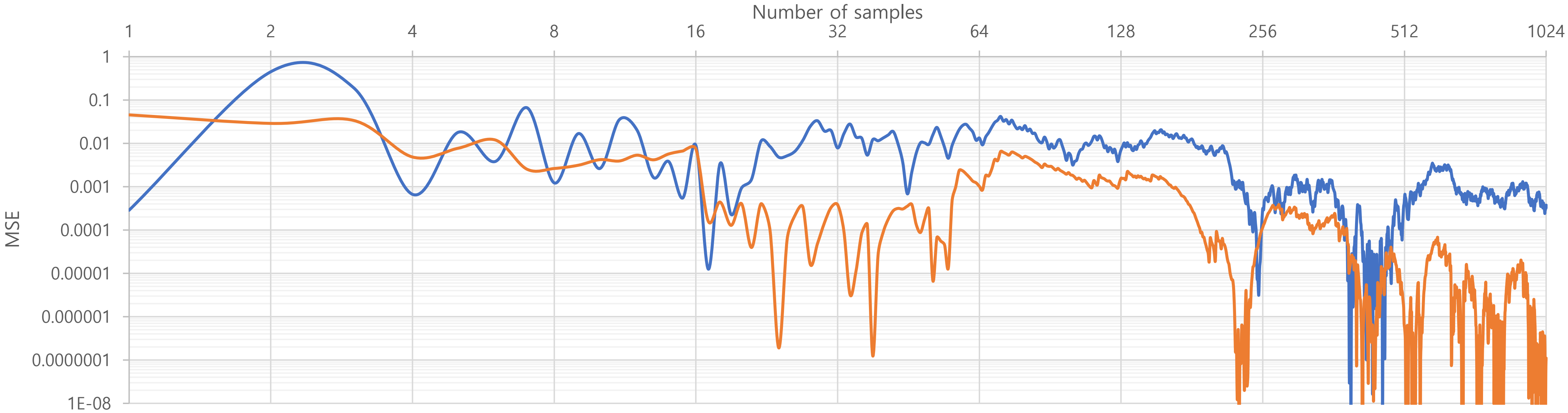
- MC integration for $\int_0^{\pi} \sin^2 x \, dx$
  - Sampling pdf p
    1. uniform distribution





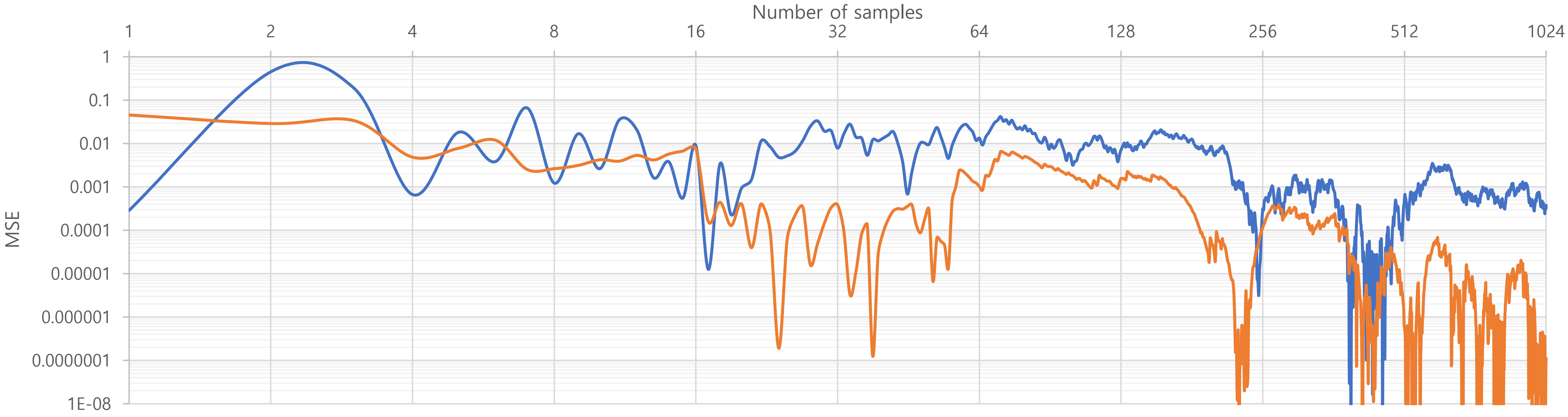
Path Guiding – 1D example

- MC integration for $\int_0^{\pi} \sin^2 x \, dx$
  - Sampling pdf p
    1. uniform distribution
    2. triangle-shaped pdf

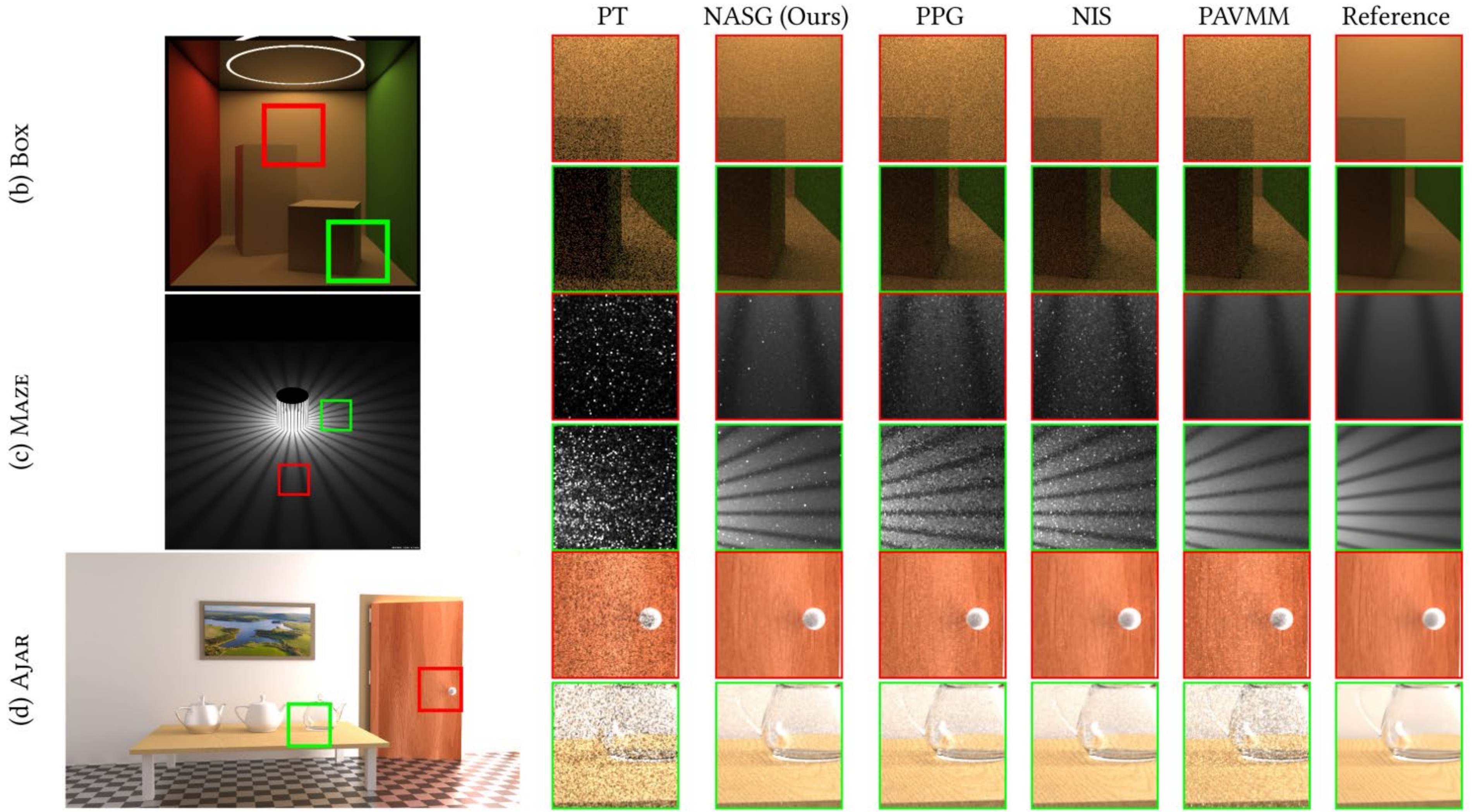


Path Guiding – 1D example

- MC integration for $\int_0^{\pi} \sin^2 x\, dx$
  - Optimal pdf: $\frac{2}{\pi}\sin^2 x \propto \sin^2 x$



Path Guiding Results



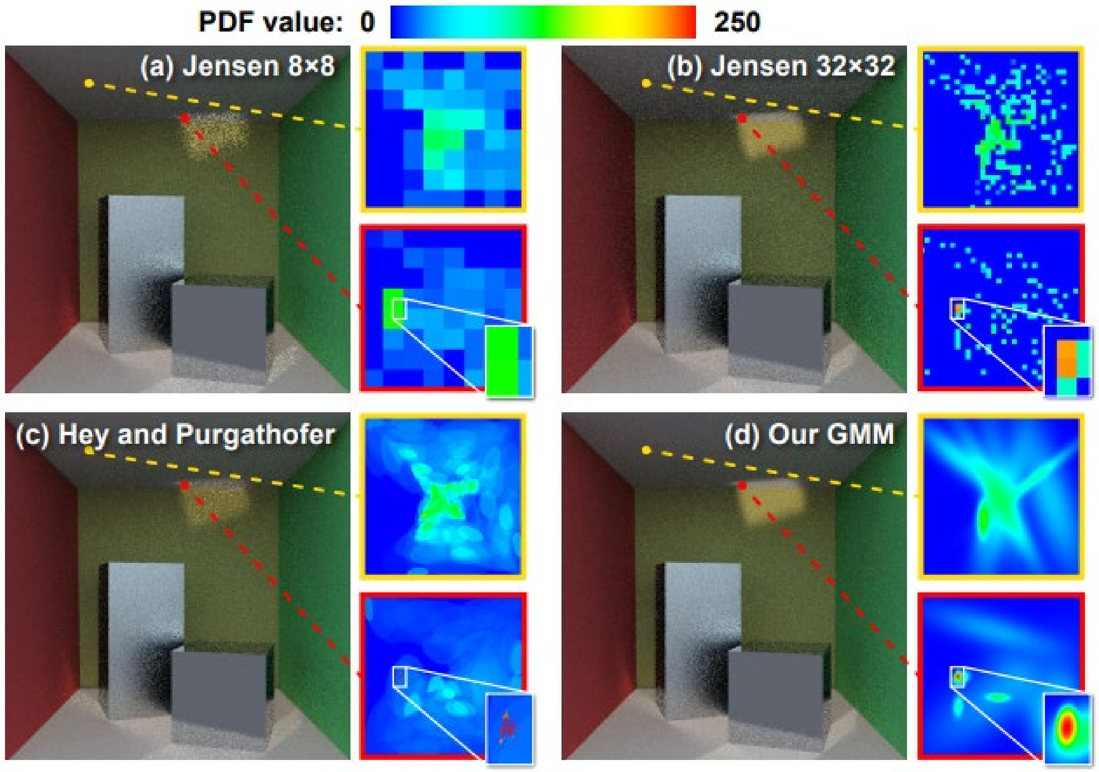
Huang, Jiawei, et al. "Online Neural Path Guiding with Normalized Anisotropic Spherical Gaussians." ACM Transactions on Graphics 43.3 (2024): 1-18.

Traditional Methods in Path Guiding

Grid-based
MM-based
Tree-based
Variance-aware

Grid-Based Path Guiding [Jensen 1995]

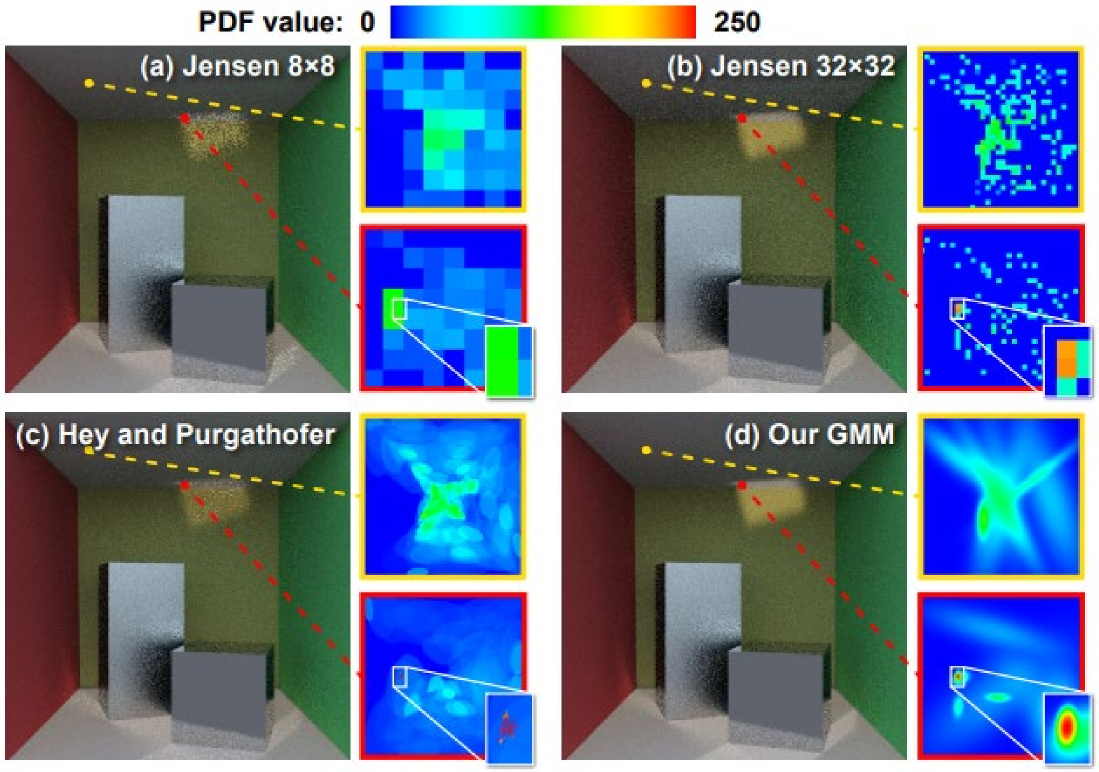
- Represent radiance fields with grid structure



Vorba, Jiří, et al. "On-line learning of parametric mixture models for light transport simulation." ACM Transactions on Graphics (TOG) 33.4 (2014): 1-11

GMM-Based Path Guiding [Vorba et al. 2014]

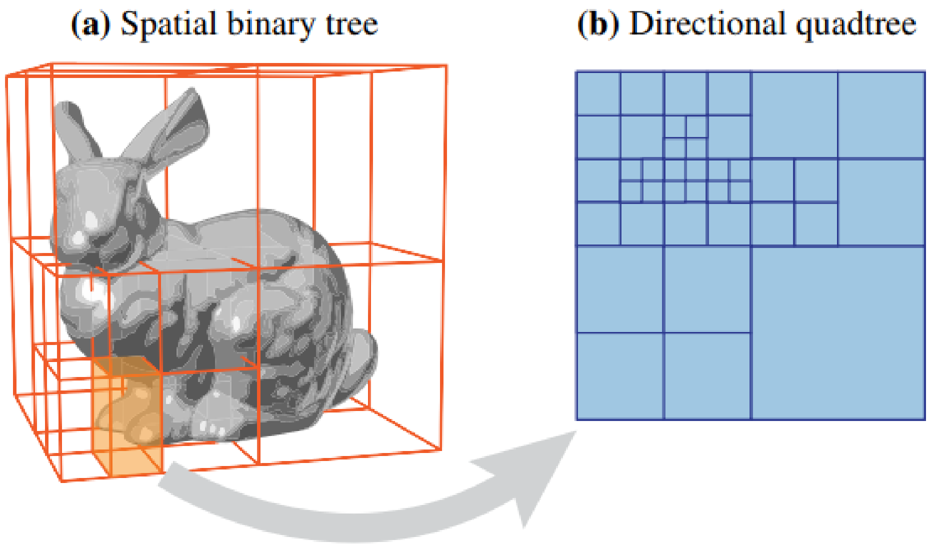
- Used Gaussian mixture model (GMM) to represent radiance fields

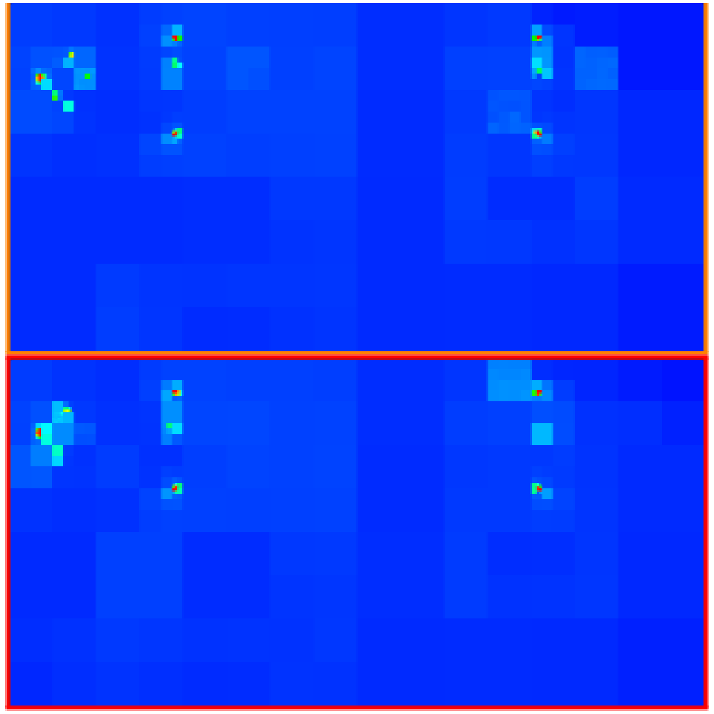


Vorba, Jiří, et al. "On-line learning of parametric mixture models for light transport simulation." ACM Transactions on Graphics (TOG) 33.4 (2014): 1-11

Tree-Based Path Guiding [Müller et al. 2017; Müller 2019]

- Used hierarchical structures
  - k-d tree for spatial subdivision
  - Each spatial leaf node contains a directional quadtree

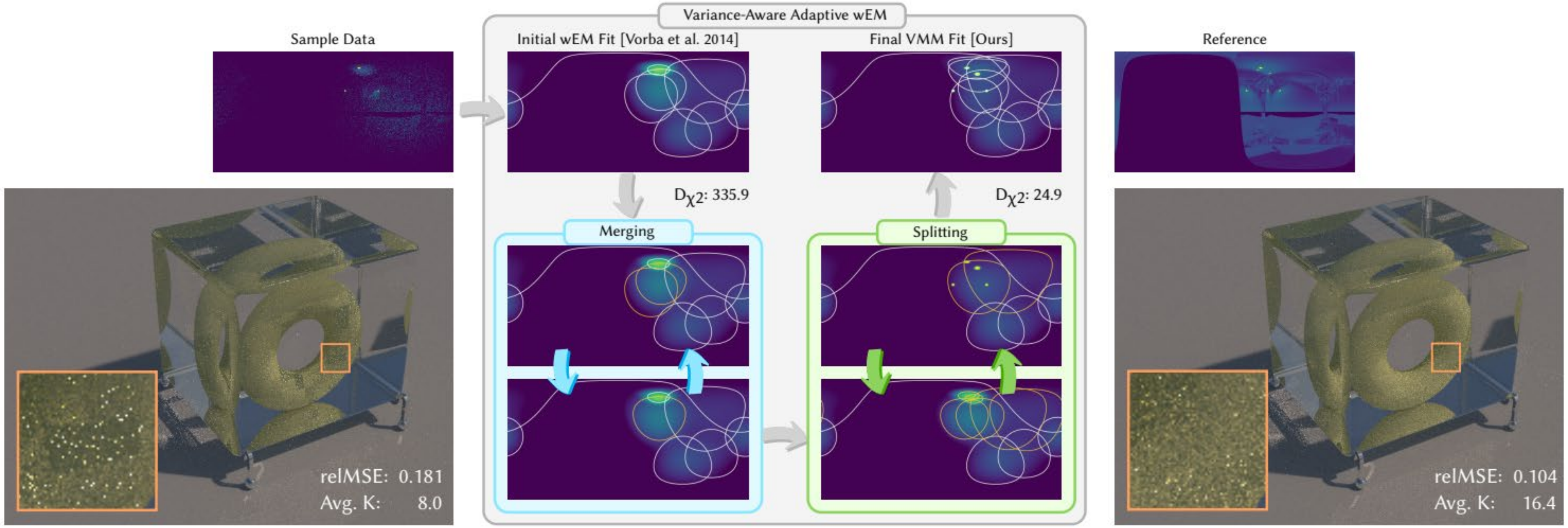




Müller, Thomas, Markus Gross, and Jan Novák. "Practical path guiding for efficient light-transport simulation." Computer Graphics Forum. Vol. 36. No. 4. 2017.

VMM-Based Path Guiding [Ruppert et al. 2020]

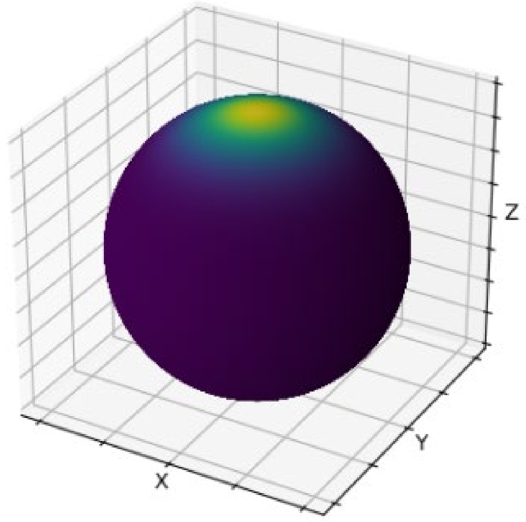
- VMM-based radiance representation
- Variance-based merge/split
- Parallax-aware representation



Ruppert, Lukas, Sebastian Herholz, and Hendrik PA Lensch. "Robust fitting of parallax-aware mixtures for path guiding." ACM Transactions on Graphics (TOG) 39.4 (2020): 147-1.

VMM-Based Path Guiding [Ruppert et al. 2020]

- Von Mises-Fischer (vMF) distribution (spherical Gaussian)
  - $v(\omega|\mu,\kappa) = \frac{\kappa}{4\pi \sinh \kappa} e^{\kappa(\mu\cdot\omega)}$
    - $\mu \in S^2$
    - $\kappa \geq 0$



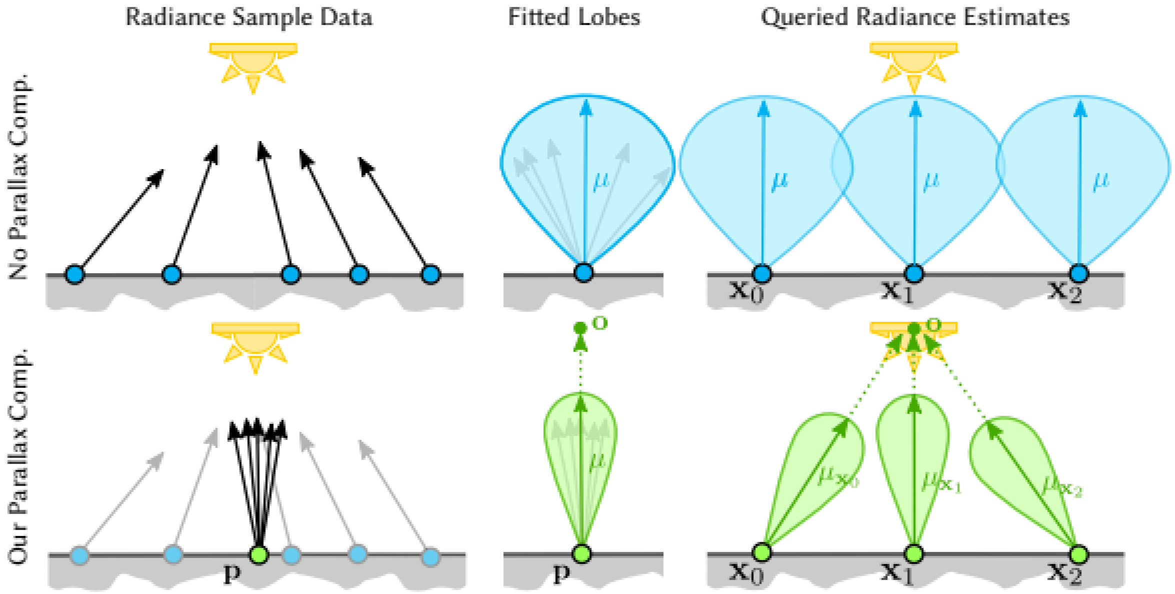
- vMF mixture model (VMM)
  - $\mathcal{V}(\omega) = \sum_{j=1}^{K} \pi_j v\left(\omega\middle|\mu_j,\kappa_j\right)$
    - $\sum_{j=1}^{K} \pi_j = 1$

VMM-Based Path Guiding [Ruppert et al. 2020]

- Overall procedure
  - Sample rays under VMM $\mathcal{V}(\omega|\Theta) = \sum_{k=1}^{K} \pi_k v(\omega|\mu_k, \kappa_k)$
    - $S = \{s_1, \cdots, s_N\}$, $s_n = \{x_n, \omega_n, p(\omega_n|x_n), \tilde{L}_i(x_n, \omega_n)\}$
  - Compute weight for each sample
    - $w_n = \frac{1}{\tilde{\Phi}(x)} \frac{\tilde{L}_i(x_n,\omega_n)}{p(\omega_n|x_n)}$, $\tilde{\Phi}(x) = \frac{1}{N} \sum_{n=1}^{N} \frac{\tilde{L}_i(x_n,\omega_n)}{p(\omega_n|x_n)}$
  - Compute sufficient statistics for updating VMM parameters
    - $r_k = \sum_{n=1}^{N} w_n \gamma_k(\omega_n) \omega_n$, $\bar{r}_k = \frac{\|r_k\|}{\sum_{n=1}^{N} w_n \gamma_k(\omega_n)}$
  - Update VMM parameters from sufficient statistics
    - $\hat{\mu}_k = \frac{r_k}{\|r_k\|}$, $\hat{\kappa}_k \approx \frac{3\bar{r}_k - \bar{r}_k^3}{1 - \bar{r}_k^2}$, $\hat{\pi}_k = \frac{\sum_{n=1}^{N} w_n \gamma_k(\omega_n)}{\sum_{j=1}^{K} \sum_{n=1}^{N} w_n \gamma_j(\omega_n)}$

VMM-Based Path Guiding [Ruppert et al. 2020]

- Parallax-aware representation for incident radiance
  - VMM is shared in each spatial region
  - Parallax causes additional estimation error
- Later, solved in MLP-based methods



Ruppert, Lukas, Sebastian Herholz, and Hendrik PA Lensch. "Robust fitting of parallax-aware mixtures for path guiding." ACM Transactions on Graphics (TOG) 39.4 (2020): 147-1.

Variance-Aware Path Guiding [Rath et al. 2020]

- Recall: If $p \propto L_i f_s \cos\theta_i$, $Var[\langle L_o(\mathbf{x}, \boldsymbol{\omega}_o)\rangle] = 0$

- Can we get exact value of $L_i(\mathbf{x}, \boldsymbol{\omega}_i)$?
  - No!

- How can we compensate estimation error for $L_i(\mathbf{x}, \boldsymbol{\omega}_i)$?

Variance-Aware Path Guiding [Rath et al. 2020]

- Goal: Finding a pdf p^* minimizing variance $Var[\langle L_o(\mathbf{x}, \boldsymbol{\omega}_o)\rangle]$

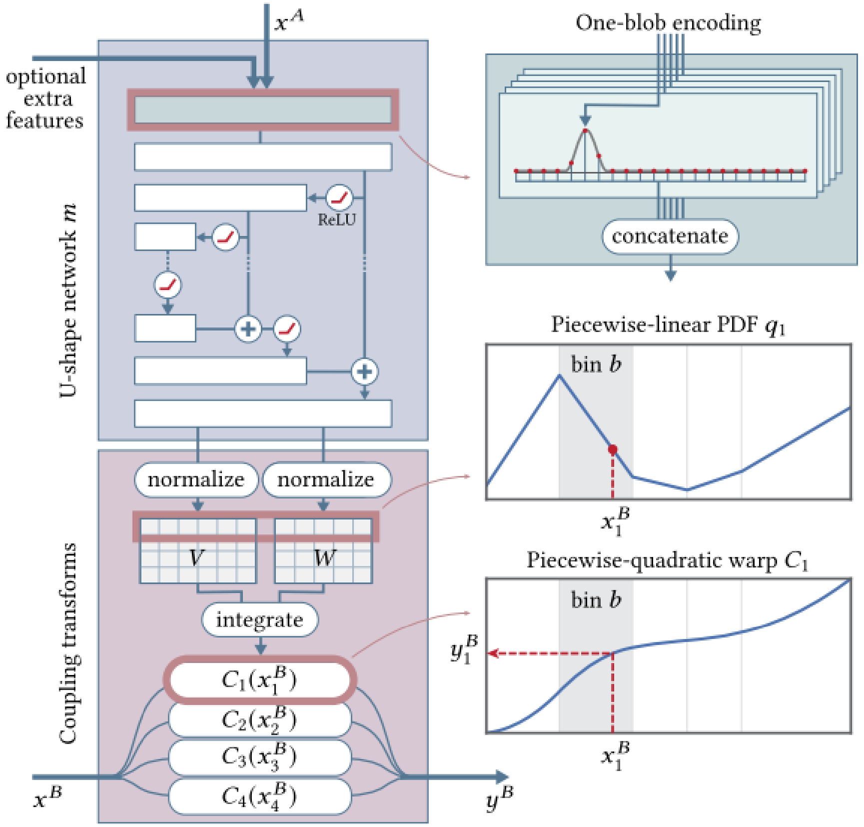
- From Euler-Lagrange equation,
  - $p^* \propto \sqrt{E[\langle L_i(\mathbf{x}, \boldsymbol{\omega}_i)\rangle^2]} f_s(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o)|\boldsymbol{\omega}_i \cdot \mathbf{n}|$
    $= \sqrt{E[\langle L_i(\mathbf{x}, \boldsymbol{\omega}_i)\rangle]^2 + Var[\langle L_i(\mathbf{x}, \boldsymbol{\omega}_i)\rangle]} f_s(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o)|\boldsymbol{\omega}_i \cdot \mathbf{n}|$

Machine Learning Approaches in Path Guiding

Normalizing flows
Offline learning
Reinforcement learning
Hierarchical CNN
Implicit representation

Neural Importance Sampling [Müller et al. 2019]

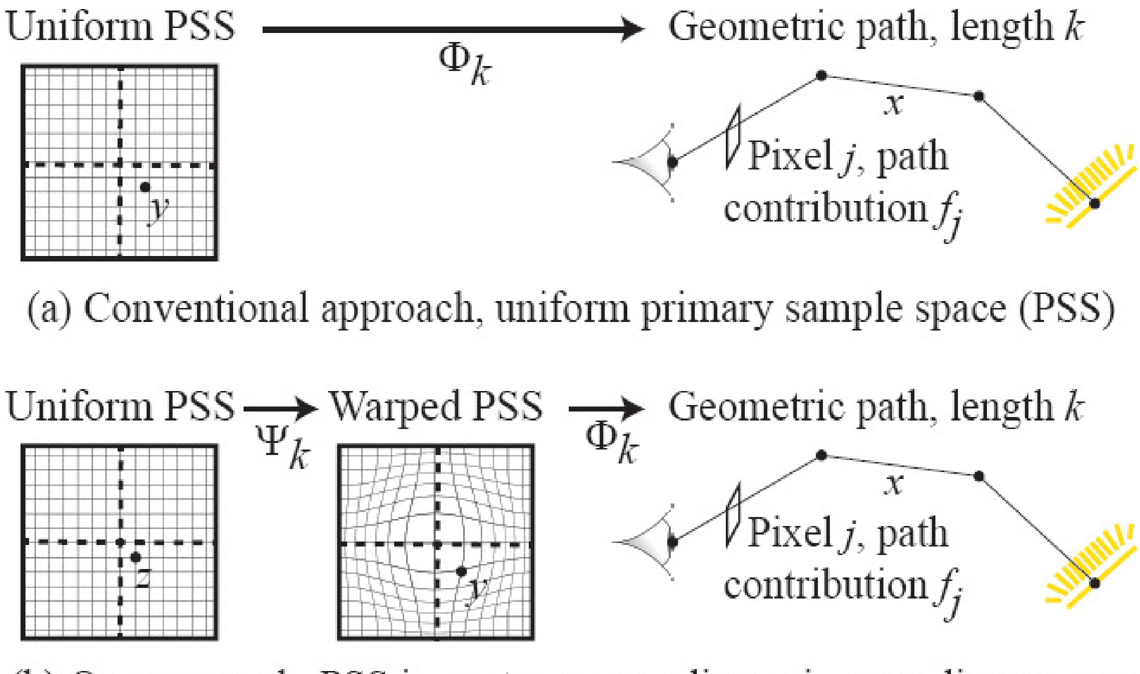
- Learns sampling functions with normalizing flows
  - Piecewise-polynomial coupling transforms



Müller, Thomas, et al. "Neural importance sampling." ACM Transactions on Graphics (ToG) 38.5 (2019): 1-19.

Primary Sample Space [Zheng and Zwicker 2019]

- Learn a bijective warping Ψ_k in primary sample space
  - Φ_k: a mapping from a canonical parametrization of paths with length k, Ω_k over $2(k+1)$-dimensional hypercube $[0,1]^{2(k+1)}$

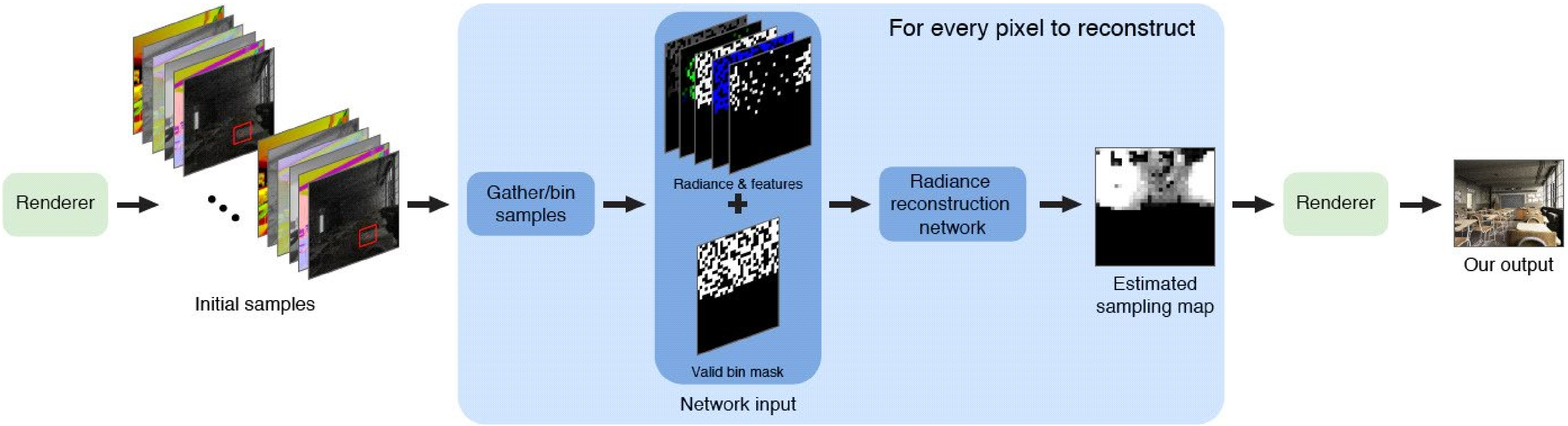


(b) Our approach, PSS importance sampling using non-linear warp

Zheng, Quan, and Matthias Zwicker. "Learning to importance sample in primary sample space." Computer Graphics Forum. Vol. 38. No. 2. 2019.

Offline Path Guiding [Bako et al. 2019]

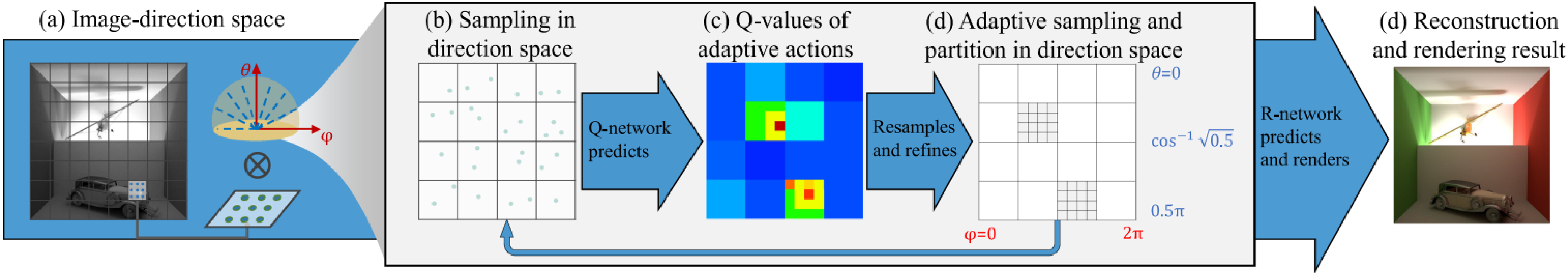
- Scene-independent method with supervised learning
- Learns incident radiance fields from local neighbor samples
  - Can be considered as denoising for incident radiance fields



Bako, Steve, et al. "Offline deep importance sampling for Monte Carlo path tracing." Computer Graphics Forum. Vol. 38. No. 7. 2019.

Reinforcement Learning [Huo et al. 2020]

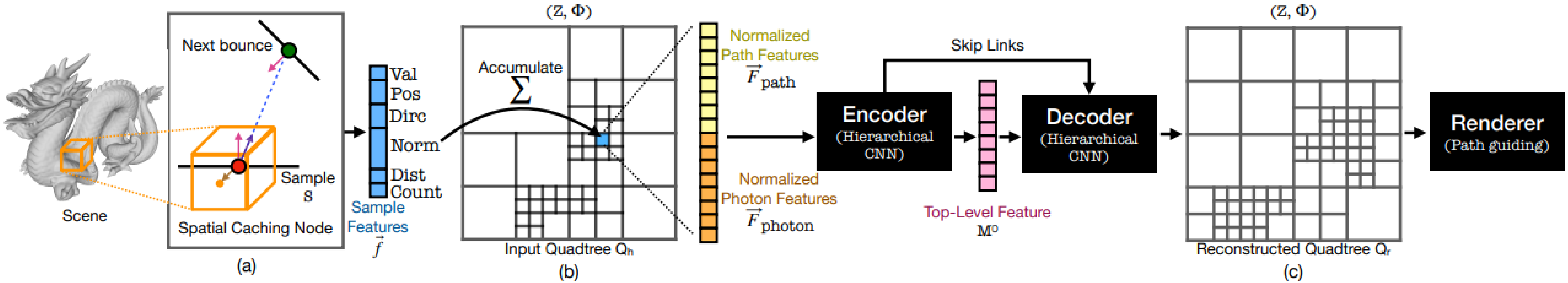
- Define two kinds of actions
  - Refine: subdivide a node
  - Resample: double the pdf value of a node
- Reward: reduced noise after radiance field denoising



Huo, Yuchi, et al. "Adaptive incident radiance field sampling and reconstruction using deep reinforcement learning." ACM Transactions on Graphics (TOG) 39.1 (2020): 1-17.

Hierarchical CNN [Zhu et al. 2021]

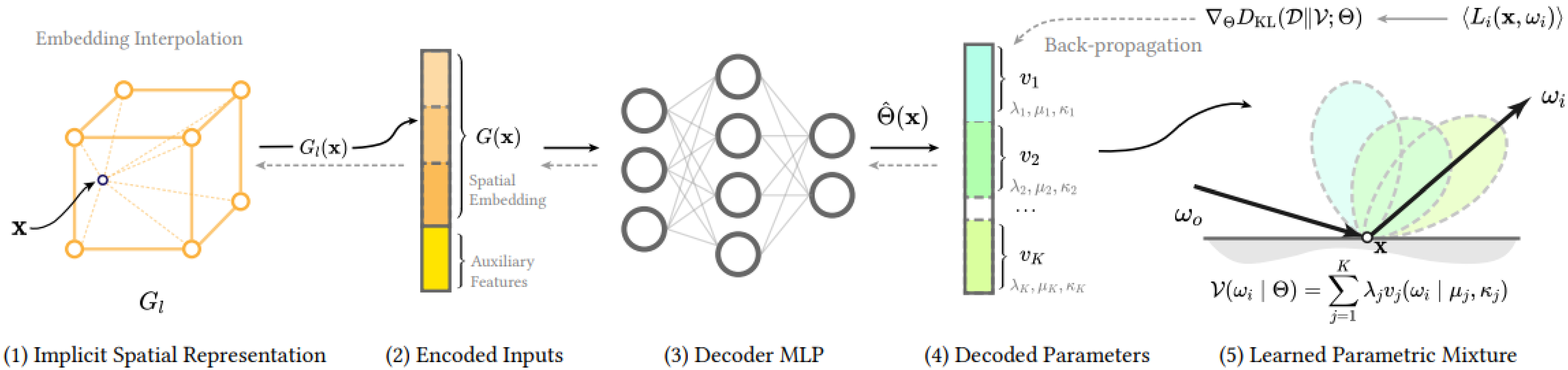
- Used hierarchical CNN to deal with quadtree



Zhu, Shilin, et al. "Hierarchical neural reconstruction for path guiding using hybrid path and photon samples." ACM Transactions on Graphics (TOG) 40.4 (2021): 1-16.

MLP-Based Representation [Dong et al. 2023]

- Estimate parameters of VMM through MLP
- Implicit representation helps solving parallax problem



Dong, Honghao, Guoping Wang, and Sheng Li. "Neural parametric mixtures for path guiding." ACM SIGGRAPH 2023 Conference Proceedings. 2023.

Thank you