Differentiable MCRT with solutions for discontinuous integrands

20223024

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Inverse rendering

Rendering

$I = f(p_0, p_1, \ldots)$

Scene parameters

Geometry, materials, lights, camera information…

Inverse Rendering

Scene parameters from images

Differentiable rendering

Scene parameters
Geometry, materials, lights, camera information…

Rendering
$I = f(p_0, p_1, \dots)$

$$\frac{\partial I}{\partial p_i}$$

Output

Target

Error(loss)

Partial derivatives
With respect to each parameter

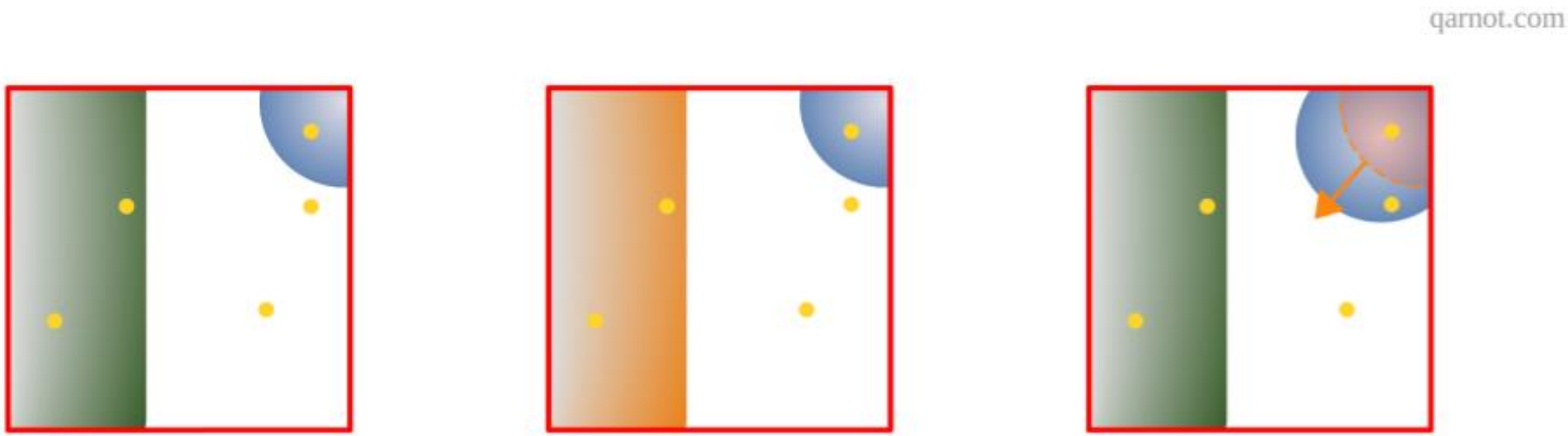
Gradient descent

Differentiable rendering

- Challenge?

Rendering integral includes visibility terms that are not differentiable at object boundaries(or discontinuities)

= **Not differentiable with respect to geometric parameters**



(4.1): no variation (4.2): color variation (4.3) : position variation

https://blog.qarnot.com/an-overview-of-differentiable-rendering/

Papers

Discontinuous integrands

Solution

Differentiable Monte Carlo Ray Tracing through Edge Sampling

LI et al., SIGGRAPH ASIA 2018

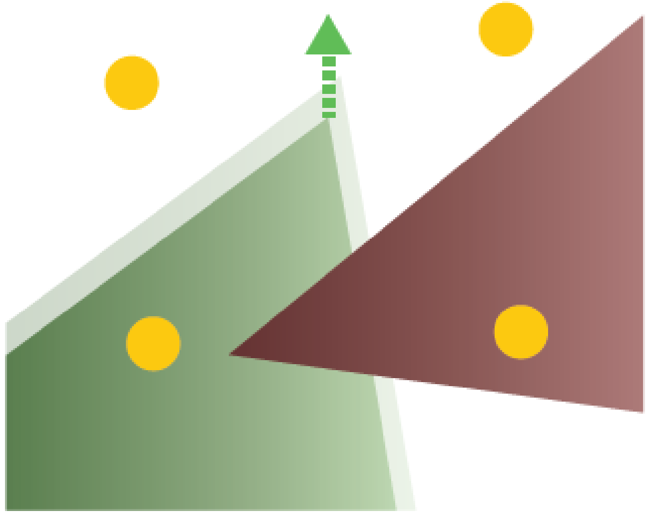
Reparameterizing Discontinuous Integrands for Differentiable Rendering

LOUBET et al., SIGGRAPH ASIA 2019

Differentiable Monte Carlo Ray Tracing through Edge Sampling

LI et al., SIGGRAPH ASIA 2018

Key idea-Edge Sampling

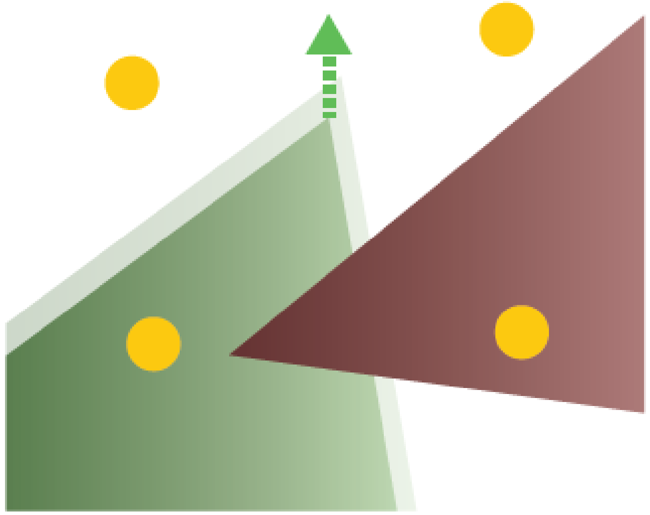


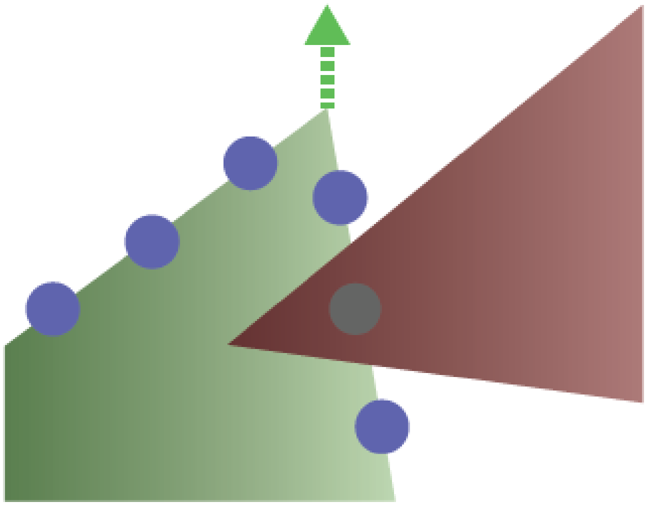
(a) area sampling

Traditional sampling

Key idea-Edge Sampling

This paper


(a) area sampling


(b) edge sampling

Traditional sampling

Mathematical Formulation

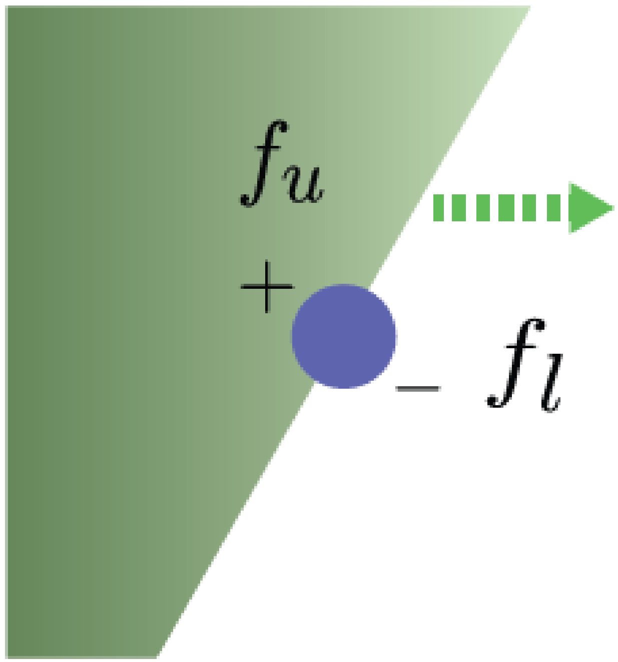
$$\nabla I = \nabla \iint f(x, y; \Phi) dxdy$$

$$\theta(\alpha(x,y)) f_u(x,y) + \theta(-\alpha(x,y)) f_l(x,y)$$

, *where*

Edge equation: $\alpha(x, y)$

Heaviside step function: $\theta()$

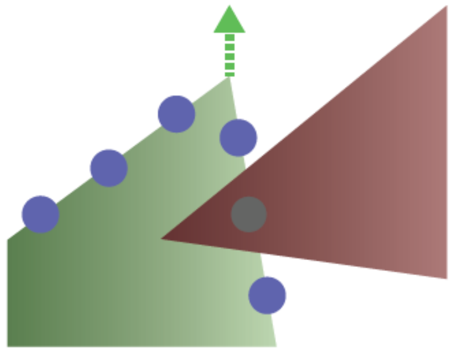


(a) half-spaces

Mathematical Formulation

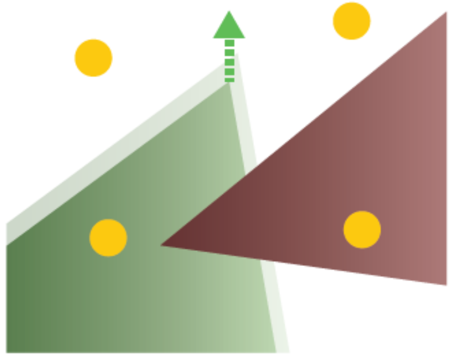
$$\iint f(x,y)dxdy = \sum_i \iint \theta(\alpha_i(x,y)) f_i(x,y)dxdy$$

$$\nabla \iint \theta(\alpha_i(x,y)) f_i(x,y)dxdy$$
$$= \iint \delta(\alpha_i(x,y))\nabla\alpha_i(x,y) f_i(x,y)dxdy$$
$$+ \iint \nabla f_i(x,y)\theta(\alpha_i(x,y))dxdy.$$



(b) edge sampling

Discontinuous region

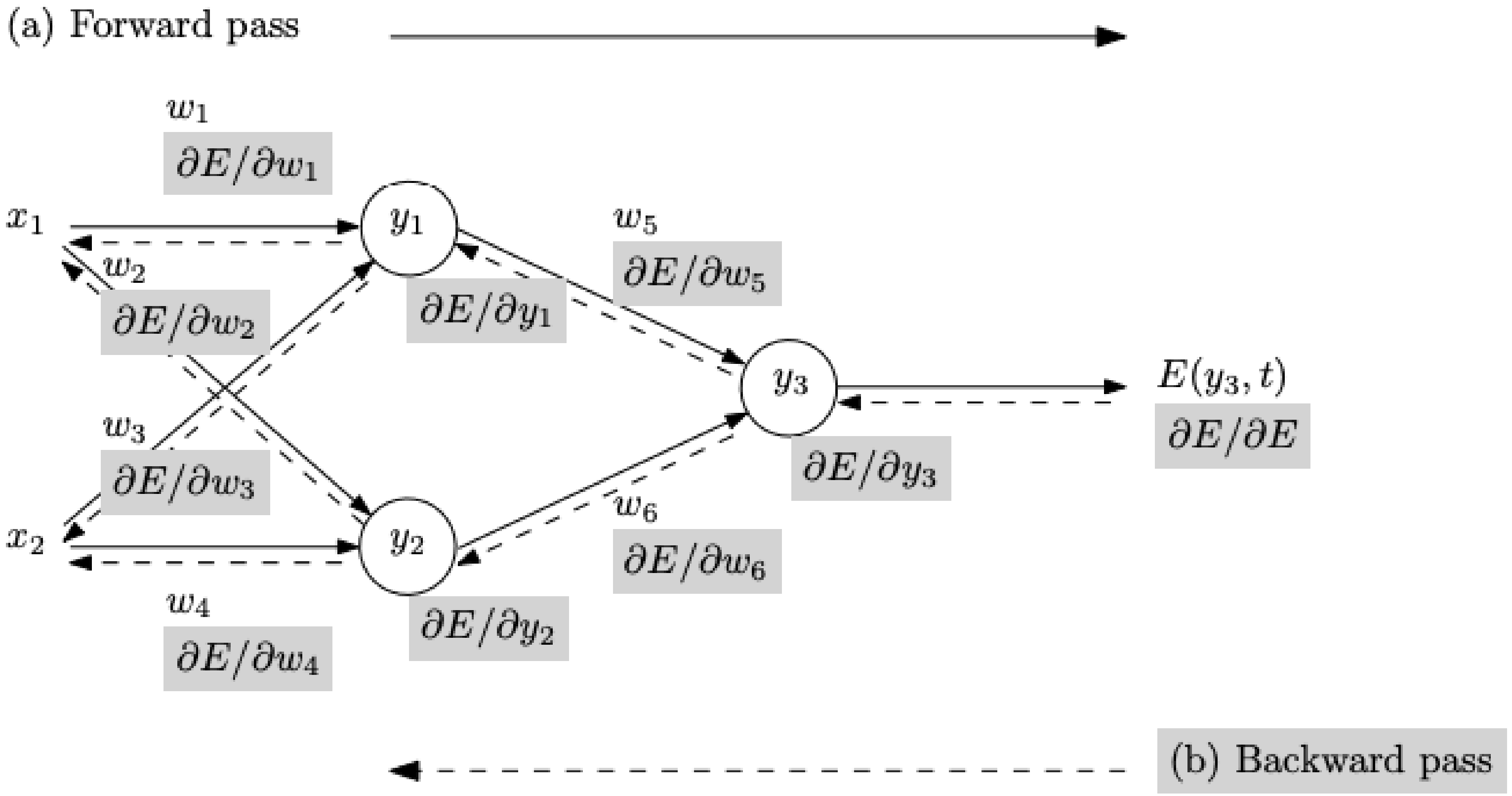


(a) area sampling

Continuous region

Derivative of continuous region

Automatic Differentiation



https://gowrishankar.info/blog/automatic-differentiation-using-gradient-tapes/

Derivative of discontinuous region

$$\iint \delta(\alpha_i(x,y))\nabla\alpha_i(x,y) f_i(x,y)\,dx\,dy$$

$$= \int_{\alpha_i(x,y)=0} \frac{\nabla\alpha_i(x,y)}{\left\|\nabla_{x,y}\alpha_i(x,y)\right\|} f_i(x,y)\,d\sigma(x,y)$$

↓

MC estimator

$$\frac{1}{N}\sum_{j=1}^{N} \frac{\|E\|\nabla\alpha_i(x_j,y_j)(f_u(x_j,y_j) - f_l(x_j,y_j))}{P(E)\left\|\nabla_{x_j,y_j}\alpha_i(x_j,y_j)\right\|}$$

Secondary visibility

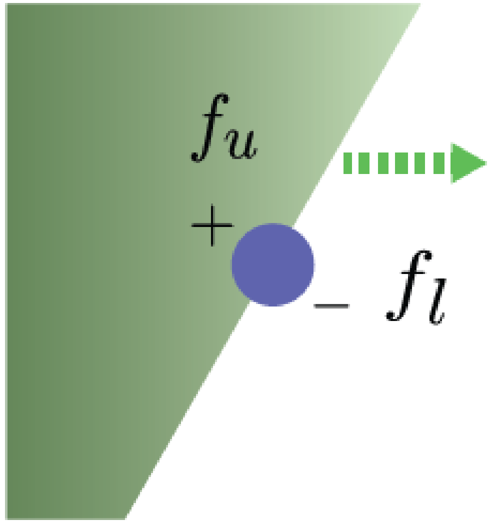
$$\nabla I = \nabla \iint f(x, y; \Phi)\,dxdy$$

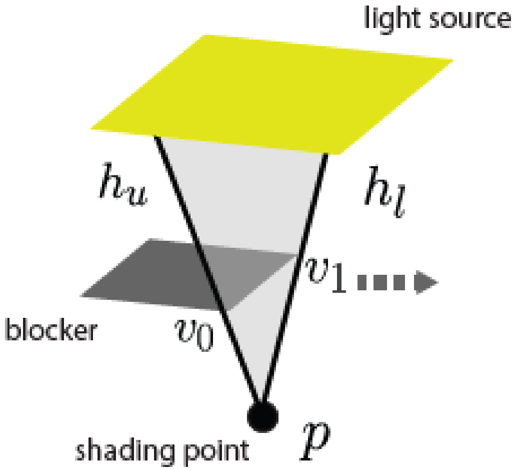
Integral for
primary visibility

$$g(p) = \int_{\mathcal{M}} h(p, m)\,dA(m)$$

Shading integral

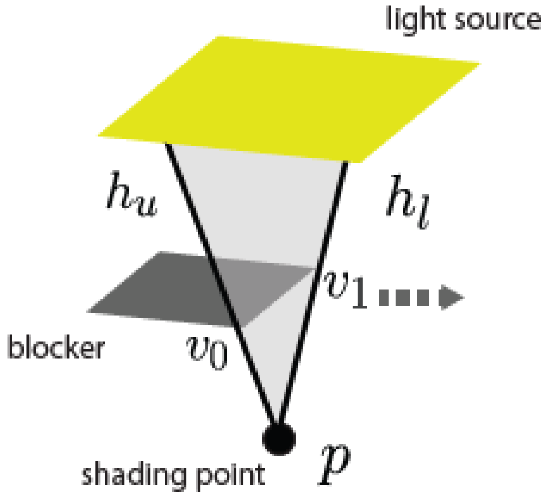
$$\theta(\alpha(p, m))h_u(p, m) + \theta(-\alpha(p, m))h_l(p, m)$$

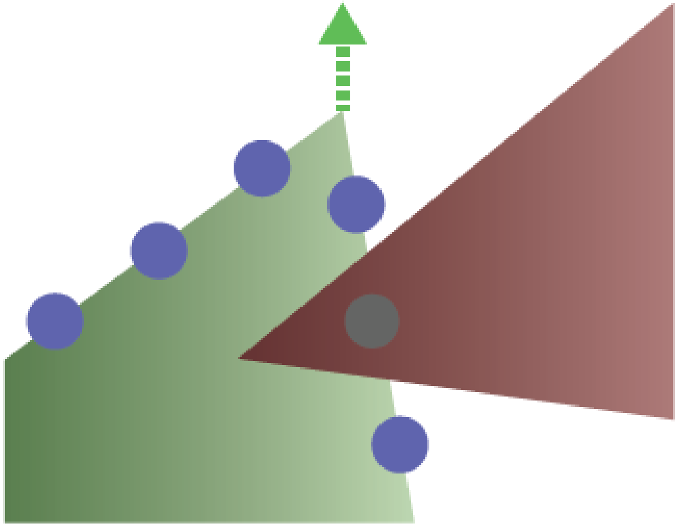




Importance edge sampling

- Need Silhouette edges, not all edges
- Edges for arbitrary viewpoint





Edge sampling

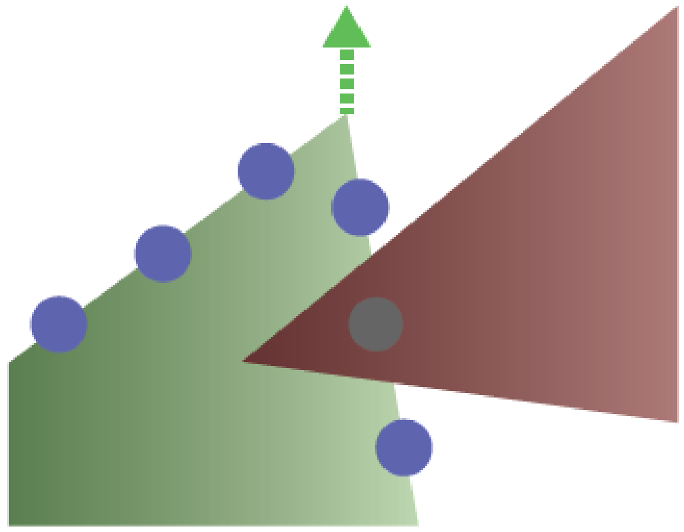
Importance edge sampling

1. Hierarchical edge sampling

- Walter et al., “Multidimensional lightcuts”, 2006
- Sander et al., “Silhouette Clipping”, SIGGRAPH 2000
- Veach et al., “Optimally Combing Sampling Techniques for Monte Carlo Rendering”, SIGGRAPH 1995

2. Importance sampling a single edge

- Heitz et al., “Real-time polygonal light shading with linearly transformed cosines”, 2016
- Heitz et al., “Linear-Light Shading with Linearly Transformed Cosines”, 2017

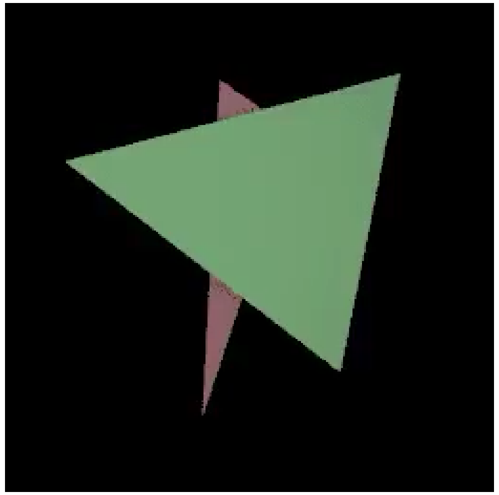


Edge sampling

Results

Optimize 6 vertices for triangles

Initial guess

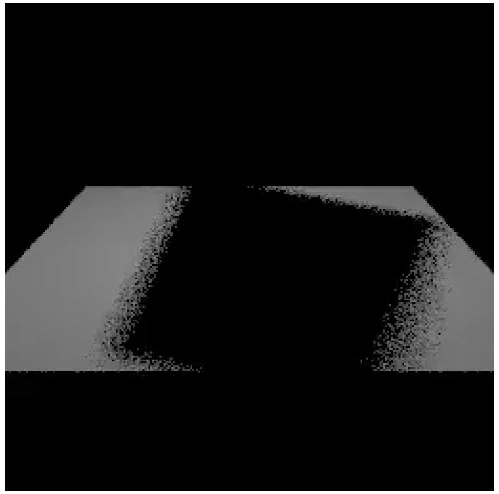


Target

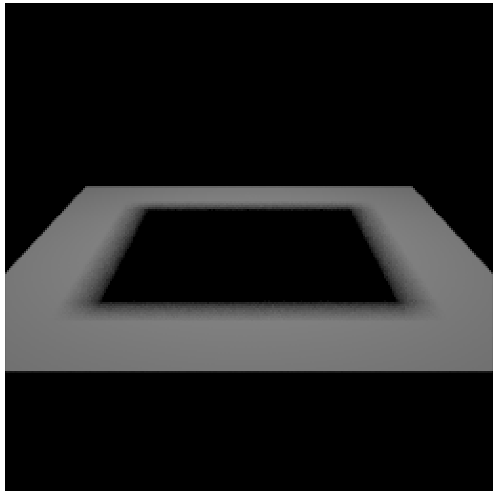
Results

Optimize blocker vertices- secondary visibility

Initial guess



Target



Limitations

- **Assumptions**

- No interpenetrating geometries and parallel edges
- Ignore shader discontinuities
- Static scenes

- **Performance**

- Edge sampling and auto differentiation are slow
- Finding all object edges and sampling them is challenging

Reparameterizing Discontinuous Integrands for Differentiable Rendering

LOUBET et al., SIGGRAPH ASIA 2019

Differentiating MC estimator(smooth case)

$$\frac{\partial}{\partial \theta} \int f(x,\theta)\,\mathrm{d}x = \int \frac{\partial}{\partial \theta} f(x,\theta)\,\mathrm{d}x$$

Leibniz integral rule

Existence and continuity of both f and its partial derivative in θ
$\Longrightarrow$ Differentiating under the integral sign

MC estimator for derivative

$$\frac{\partial I}{\partial \theta} \approx \frac{1}{N} \sum \frac{\partial}{\partial \theta} \frac{f(x_i,\theta)}{p(x_i)} = \frac{\partial E}{\partial \theta}$$

The case of non-differentiable

$$\frac{\partial}{\partial\theta}\int f(x,\theta)\,\mathrm{d}x = \int \frac{\partial}{\partial\theta} f(x,\theta)\,\mathrm{d}x$$

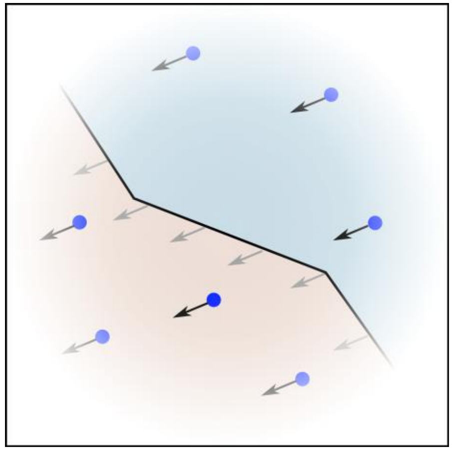
Approach: reparametrizing integrals

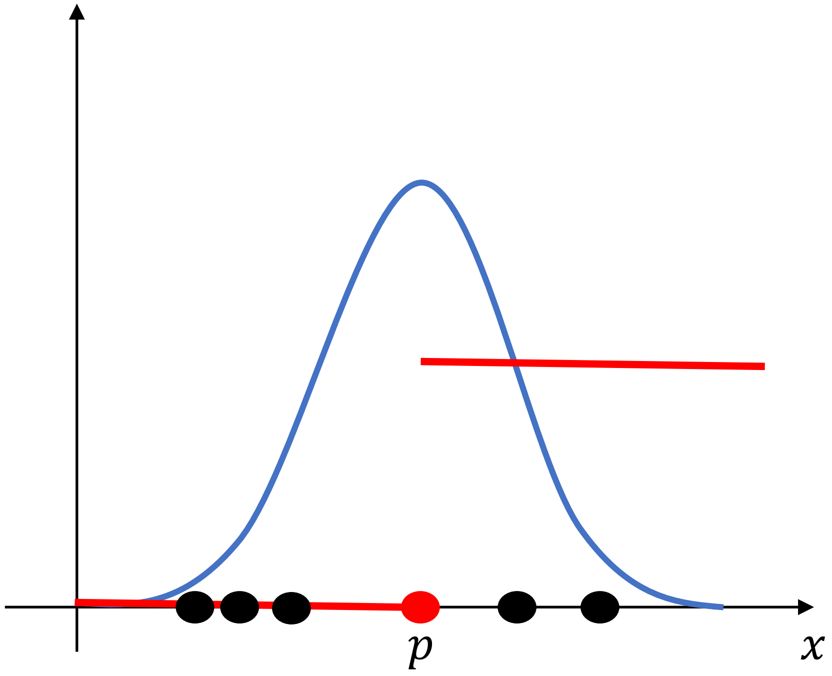
$$I = \int k(x)\mathbb{1}_{x>p}\,dx$$

$$\frac{\partial I}{\partial p} = ?$$

Change of variable: $X = x - p$

$$I = \int k(X+p)\mathbb{1}_{X>0}\,dx$$



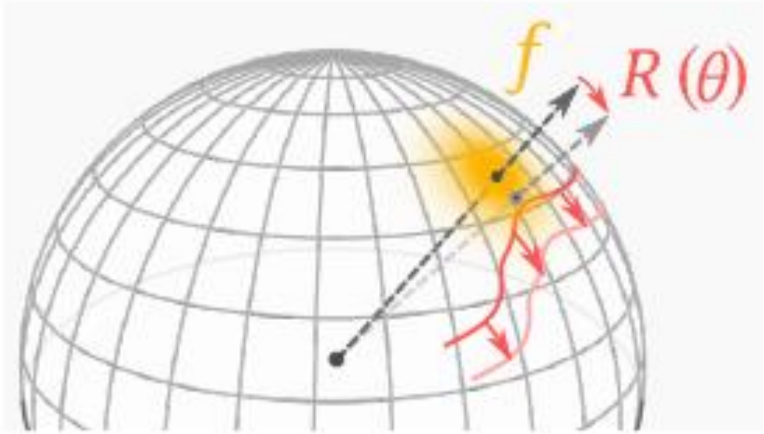


- Same value of the integral
- But different partial derivatives for MC samples

Method: removing discontinuities using rotations

Key Assumption:
Integrands have small angular support
-> Discontinuities happen on the silhouette of a single object

$$I = \int_{S^2} f(\omega, \theta)\, \mathrm{d}\omega = \int_{S^2} f(R(\omega, \theta), \theta)\, \mathrm{d}\omega$$



(a) Differentiable rotation of directions

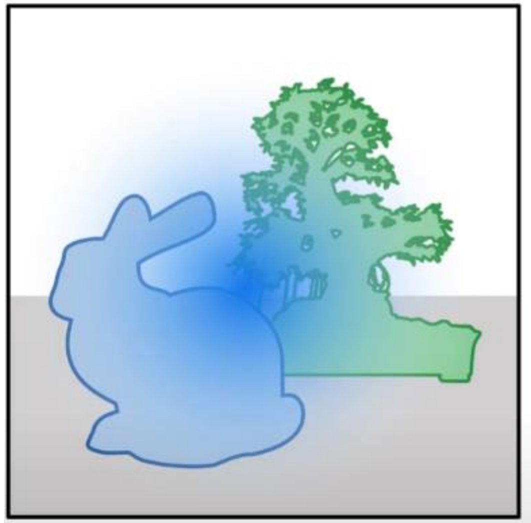
MC estimator

$$E = \frac{1}{N} \sum \frac{f(R(\omega_i, \theta), \theta)}{p(\omega_i, \theta_0)} \approx I$$

Integral with small support

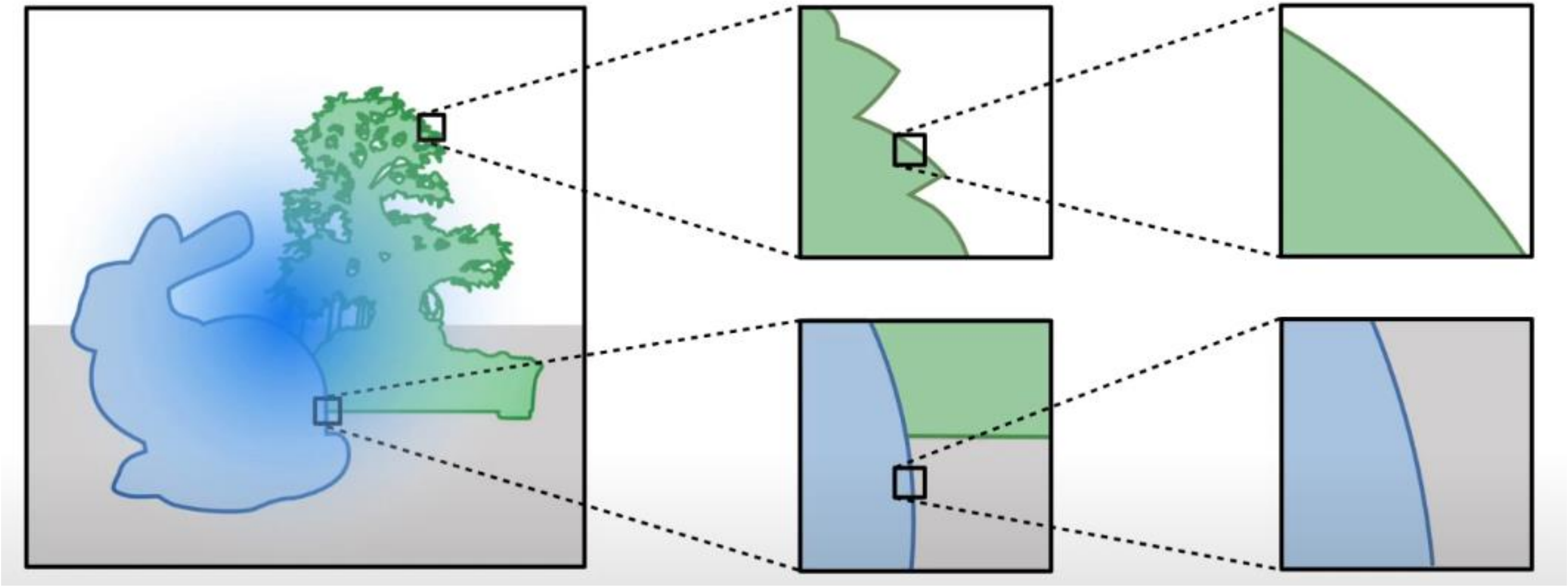
- Previous method is for discontinuities caused by silhouette of a single object
- Then, how can we deal with large support?

Small support



Large support

Integrals with large support



Zoon in enough to make the integrand locally differentiable

Integrals with large support

$\int$  $\mathrm{d}\omega = \int$  $\mathrm{d}\omega$

Integral of a function **Integral of a convolution of this function**

Integrals with large support

$$\int_{S^2} f(\omega)\,\mathrm{d}\omega = \int_{S^2}\int_{S^2} f(\mu)\,k(\mu,\omega)\,\mathrm{d}\mu\,\mathrm{d}\omega$$,where $$\int_{S^2} k(\mu,\omega)\,\mathrm{d}\mu = 1. \quad \forall \omega \in S^2$$

Set concentration parameter for k to have small angular support

MC estimator

$$I \approx E = \frac{1}{N}\sum \frac{f(R_i(\mu_i,\theta),\theta)\,k(R_i(\mu_i,\theta),\omega_i(\theta),\theta)}{p(\omega_i(\theta),\theta)\,p_k(\mu_i)}$$

Determining suitable rotations

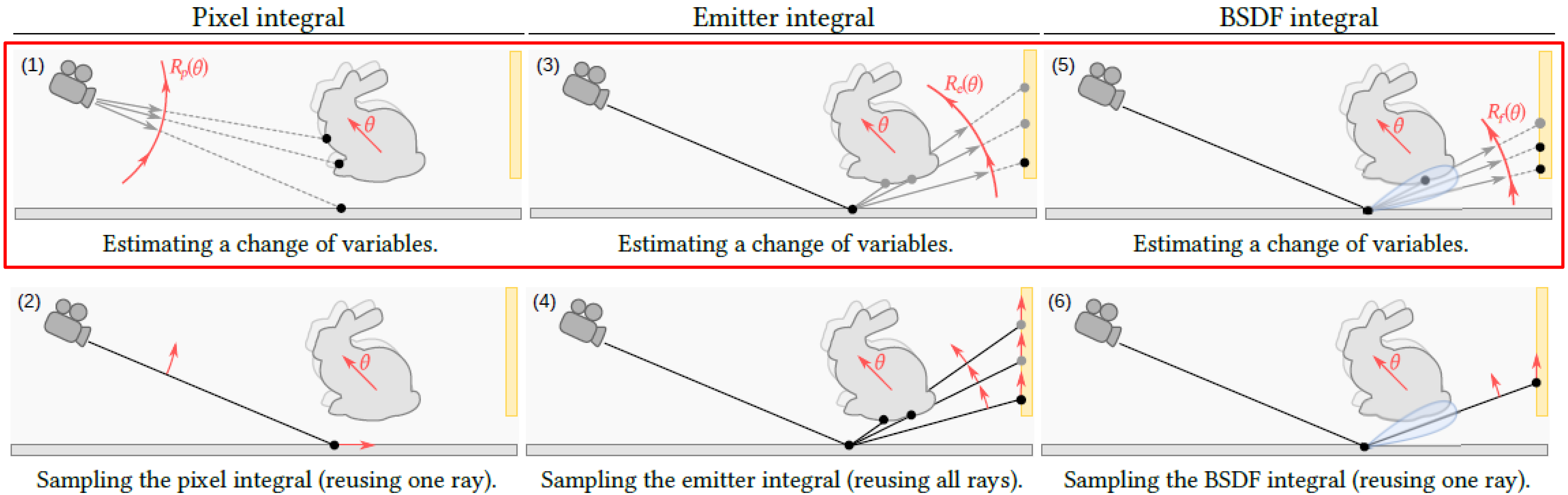
Displacement of discontinuities w.r.t infinitesimal perturbations of scene parameters

‖

Displacement of other positions on the associated object

Determining suitable rotations

- Sample rays within the support of the integrand against the scene geometry
- Select a single point for rotation matrix(**heuristic in Appendix C**)



Rotation matrix

$\boldsymbol{\omega_P}(\boldsymbol{\theta})$:Projection of the selected point onto domain S^2
$\boldsymbol{\theta_0}$: Direction associated with the current parameter configuration

$$R(\theta_0)\,\omega = \omega,\ \forall\omega \in S^2 \quad \text{and} \quad \frac{\partial}{\partial\theta}R(\theta)\,\omega_{P0} = \frac{\partial}{\partial\theta}\omega_P(\theta)$$

Closed-form expression

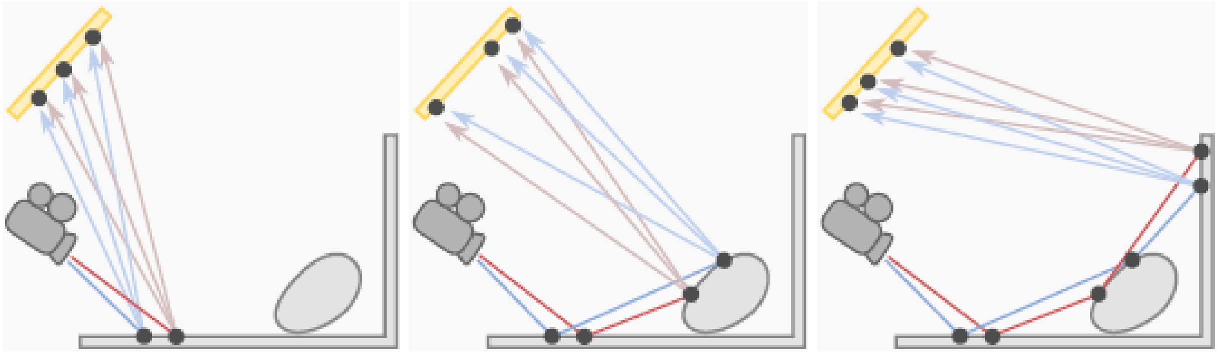
$$R = \alpha I + [\beta]_x + \frac{1}{1+\alpha}\beta^T\beta \quad \text{, where } R\omega_a = \omega_b$$

with $\alpha = \omega_a \cdot \omega_b$, $\beta = \omega_a \times \omega_b$ and

$$[u]_x = \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix}$$

Variance reduction

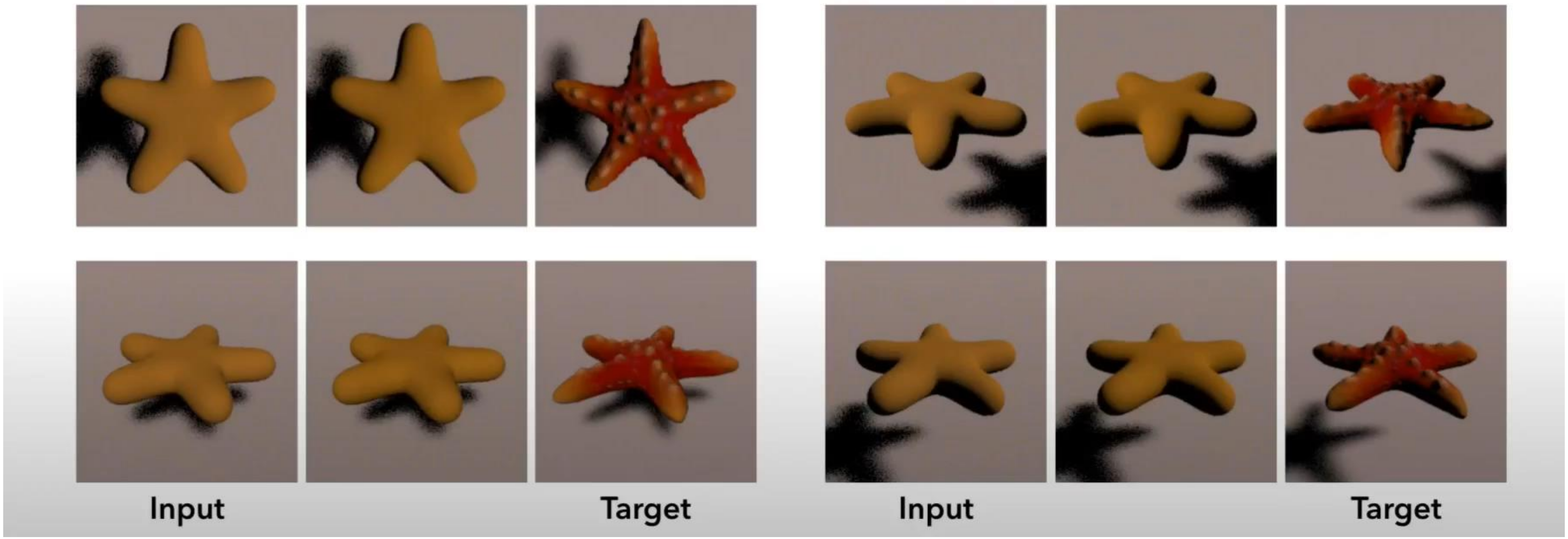
Control variate method + α



(a) Bounce 1. (b) Bounce 2. (c) Bounce 3.

Results

Synthetic example, optimised using 5 views (4 are shown)



Limitations

- The method relies on approximation that introduce bias
- It can be higher in some pixels and configurations