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# CS580: Radiometry and Rendering Equation

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Course URL:

<http://sgvr.kaist.ac.kr/~sungeui/GCG/>

**KAIST**



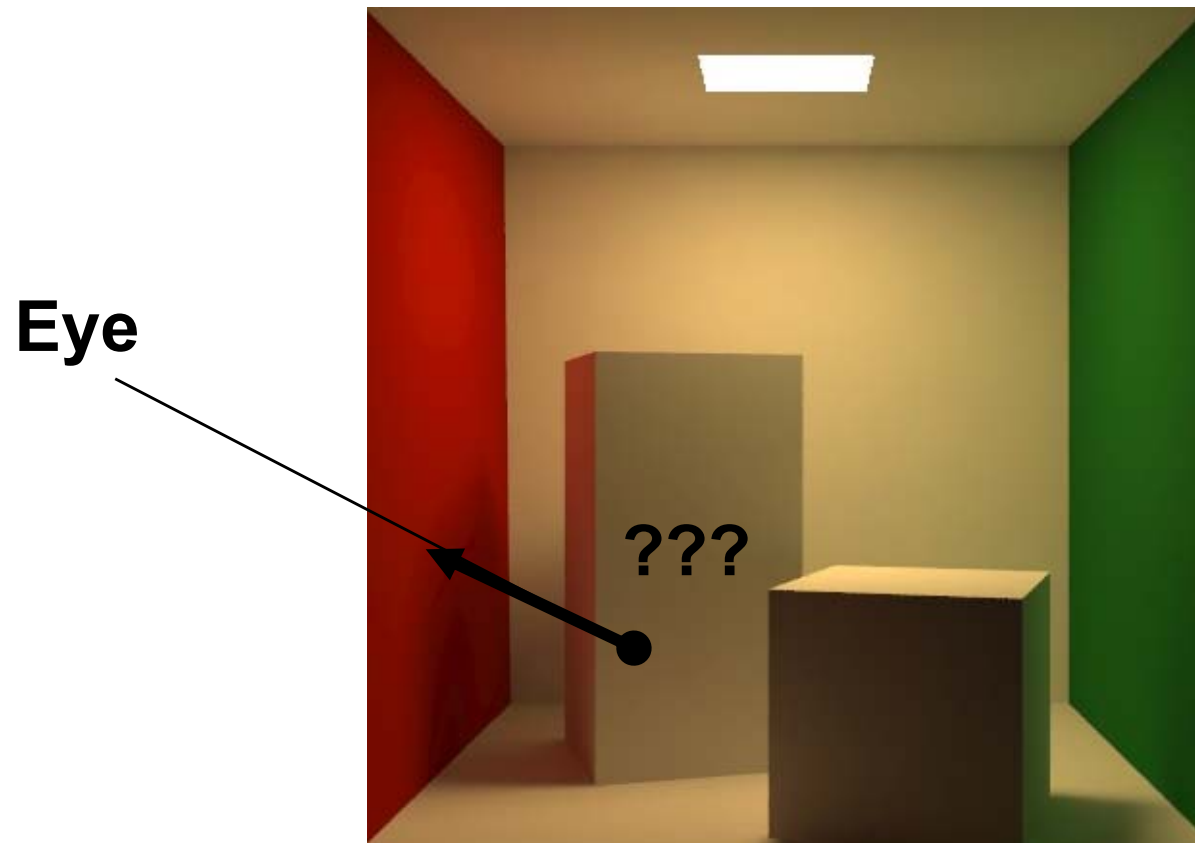
# Class Objectives (Ch. 12 and 13)

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- **Know terms of:**
  - **Hemispherical coordinates and integration**
  - **Various radiometric quantities (e.g., radiance)**
  - **Basic material function, BRDF**
  - **Understand the rendering equation**
- **Radiometric quantities**
  - **Briefly touched here**
  - **Refer to my book, if you want to know more**

# Motivation

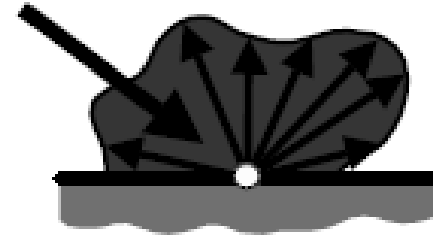
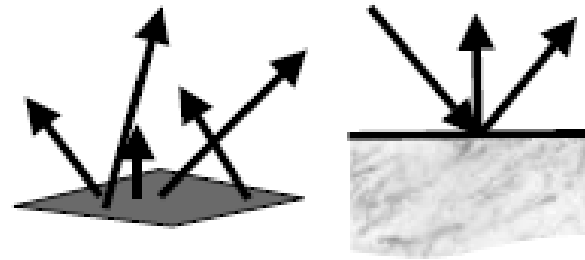
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# Light and Material Interactions

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- **Physics of light**
- **Radiometry**
- **Material properties**
  
- **Rendering equation**



From kavita's slides

# Models of Light

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- **Quantum optics**
  - **Fundamental model of the light**
  - **Explain the dual wave-particle nature of light**
- **Wave model**
  - **Simplified quantum optics**
  - **Explains diffraction, interference, and polarization**
- **Geometric optics**
  - **Most commonly used model in CG**
  - **Size of objects  $\gg$  wavelength of light**
  - **Light is emitted, reflected, and transmitted**



# Radiometry and Photometry

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- **Photometry**

- Quantify the perception of light energy

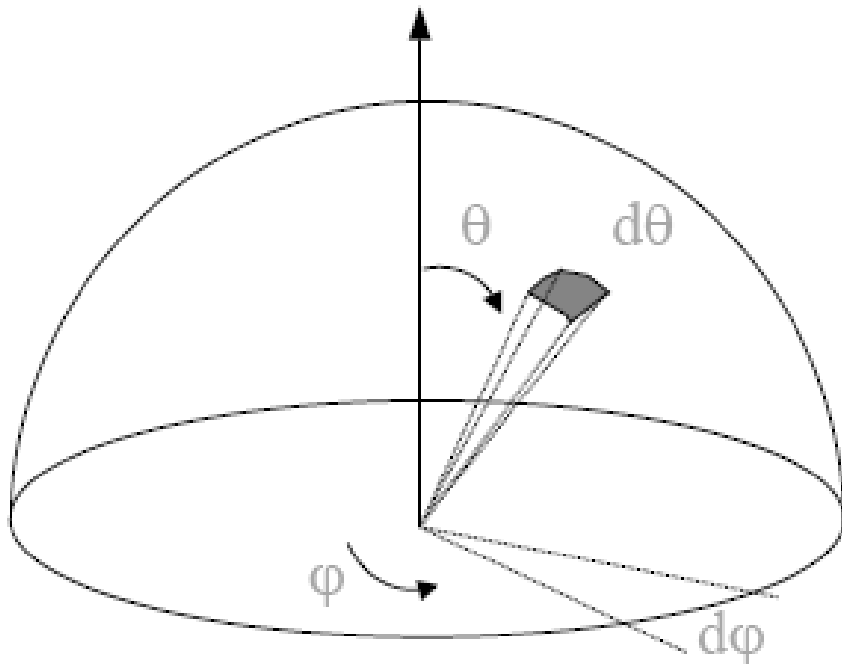
- **Radiometry**

- **Measurement of light energy: critical component for photo-realistic rendering**
- **Light energy flows through space, and varies with time, position, and direction**
- **Radiometric quantities: densities of energy at particular places in time, space, and direction**
- **Briefly discussed here; refer to my book**

# Hemispheres

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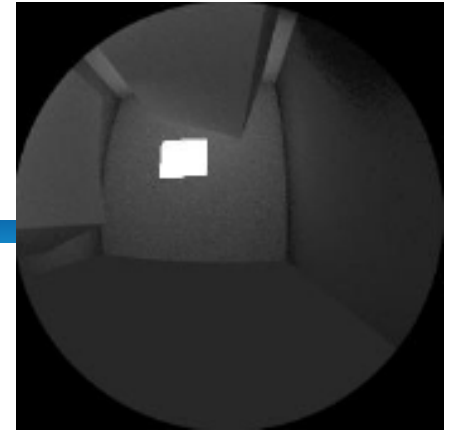
- **Hemisphere**
  - **Two-dimensional surfaces**
- **Direction**
  - **Point on (unit) sphere**



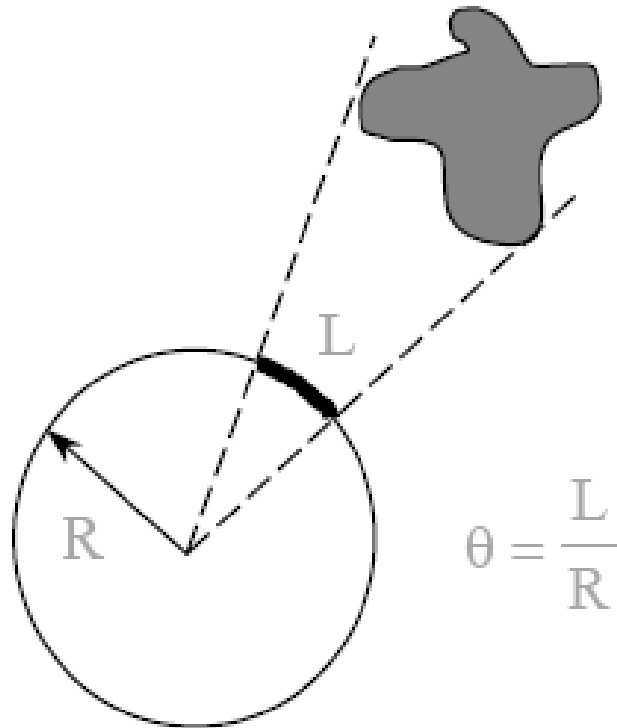
$$\theta \in \left[0, \frac{\pi}{2}\right]$$
$$\varphi \in [0, 2\pi]$$

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# Solid Angles



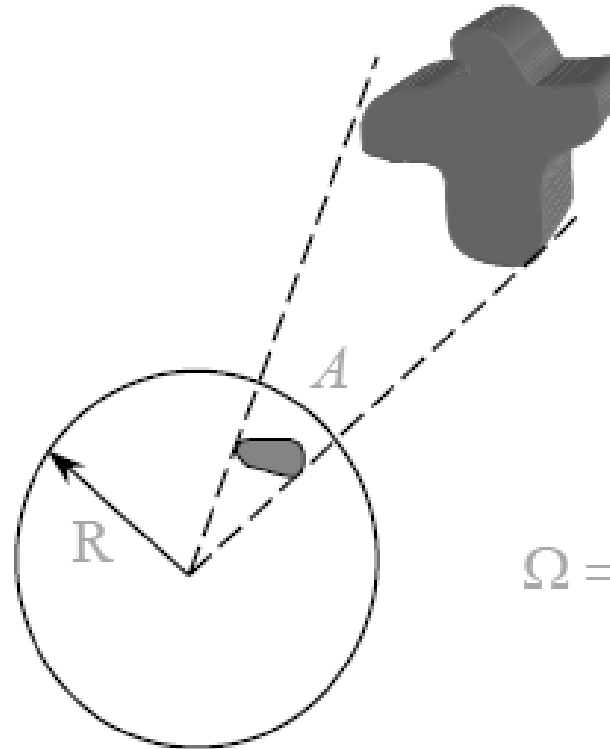
2D



$$\theta = \frac{L}{R}$$

**Full circle  
= 2pi radians**

3D



$$\Omega = \frac{A}{R^2}$$

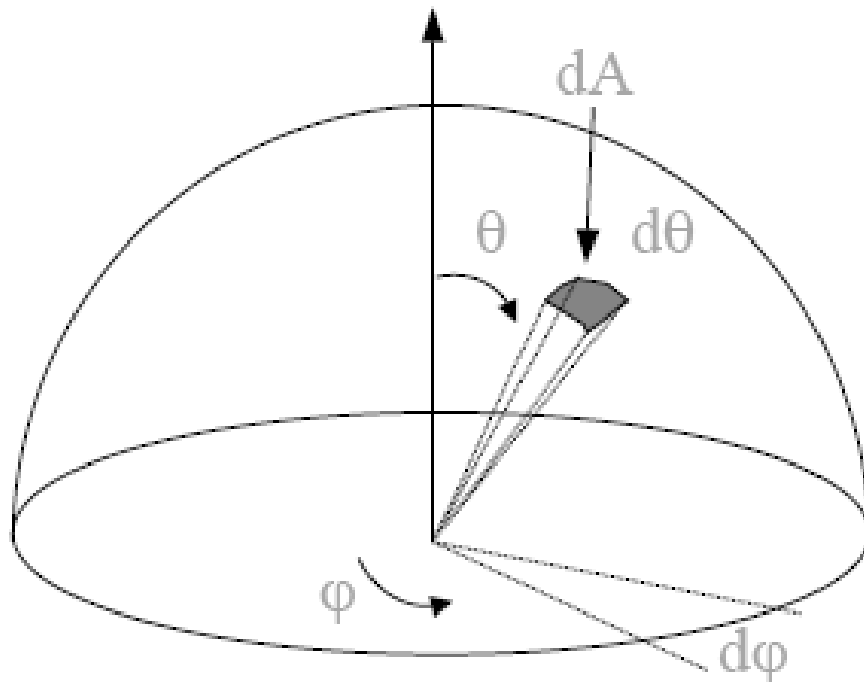
**Full sphere  
= 4pi steradians**

**View on the  
hemisphere**



# Hemispherical Coordinates

- Direction,  $\ominus$ 
  - Point on (unit) sphere

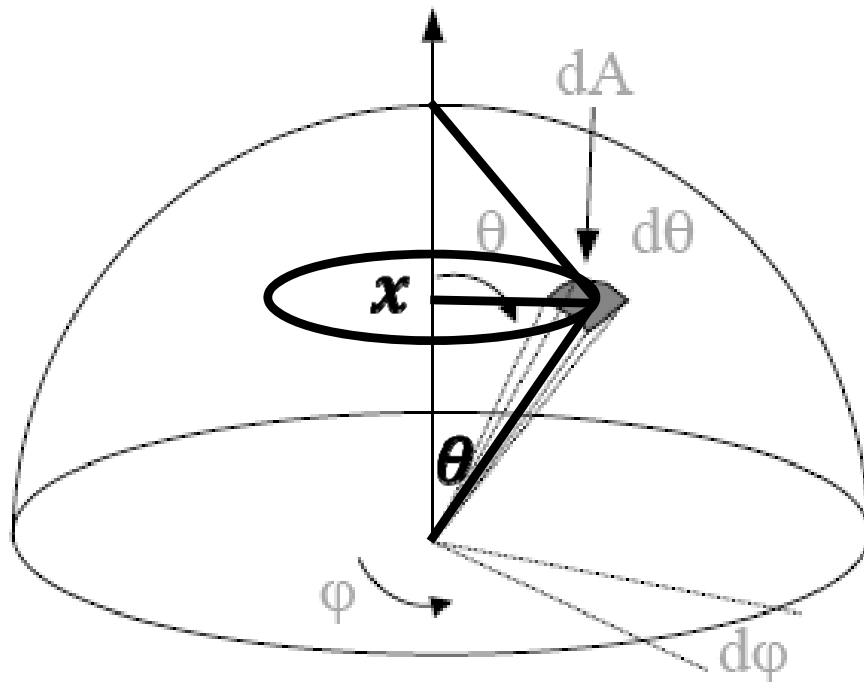


$$dA = (r \sin \theta d\phi)(r d\theta)$$

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# Hemispherical Coordinates

- Direction,  $\ominus$ 
  - Point on (unit) sphere



$$\sin \theta = \frac{x}{r},$$
$$x = r \sin \theta$$

$$dA = (r \sin \theta d\phi)(r d\theta)$$

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# Hemispherical Coordinates

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- **Differential solid angle**

$$d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\varphi$$

# Hemispherical Integration

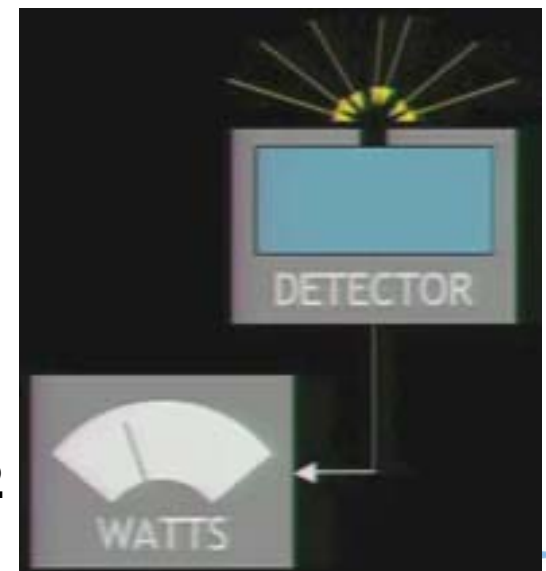
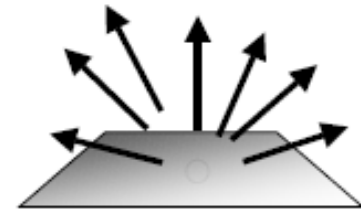
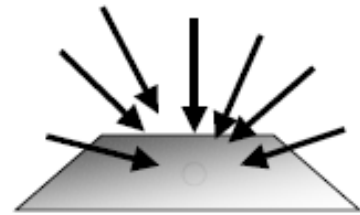
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- **Area of hemisphere:**

$$\begin{aligned}\int_{\Omega_x} d\omega &= \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin\theta d\theta \\ &= \int_0^{2\pi} d\varphi [-\cos\theta]_0^{\pi/2} \\ &= \int_0^{2\pi} d\varphi \\ &= 2\pi\end{aligned}$$

# Irradiance

- **Incident radiant power per unit area ( $dP/dA$ )**
  - Area density of power
- **Symbol:  $E$ , unit:  $W/m^2$** 
  - Area power density existing a surface is called radiance exitance ( $M$ ) or radiosity ( $B$ )
- **For example**
  - A light source emitting 100 W of area  $0.1 m^2$
  - Its radiant exitance is  $1000 W/m^2$



# Radiance

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- **Radiant power at  $x$  in direction  $\theta$** 
  - $L(x \rightarrow \Theta)$  : 5D function
    - Per unit area
    - Per unit solid angle

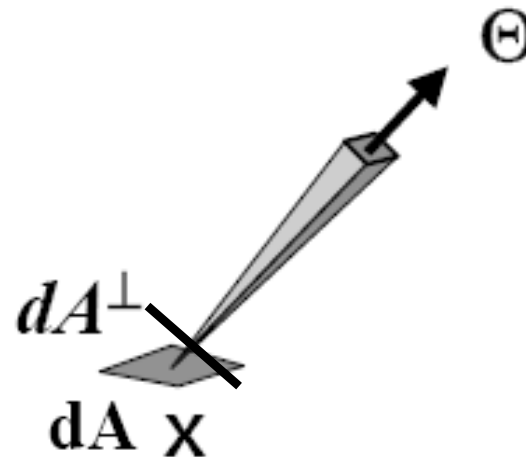


- **Important quantity for rendering**

# Radiance

- **Radiant power at  $x$  in direction  $\Theta$** 
  - $L(x \rightarrow \Theta)$  : 5D function
    - **Per unit area**
    - **Per unit solid angle**

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^\perp d\omega_\Theta}$$

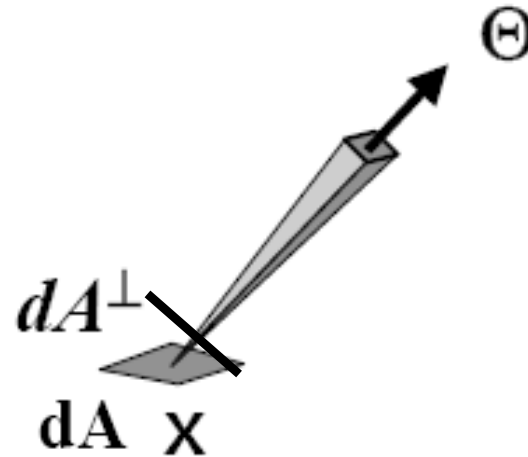


- **Units: Watt / (m<sup>2</sup> sr)**
- **Irradiance per unit solid angle**
- **2<sup>nd</sup> derivative of P**
- **Most commonly used term**

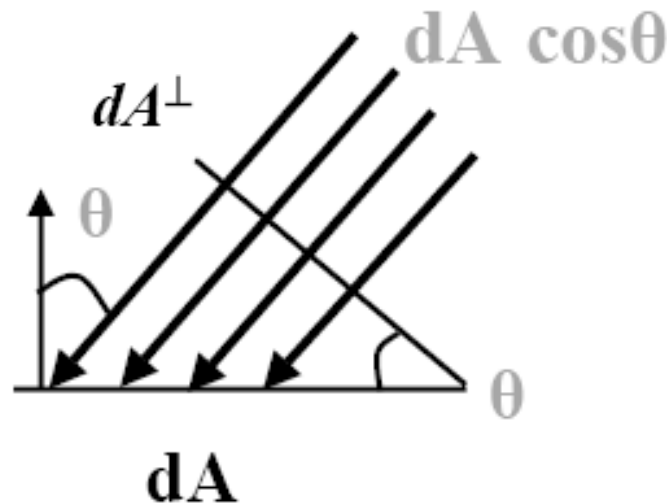
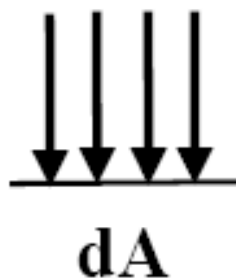
# Radiance: Projected Area

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^\perp d\omega_\Theta}$$

$$= \frac{d^2 P}{d\omega_\Theta dA \cos \theta}$$



- Why per unit projected surface area

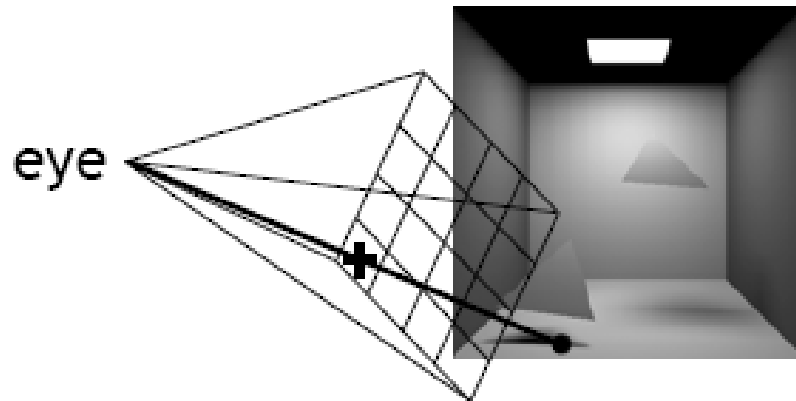




# Sensitivity to Radiance

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- Responses of sensors (camera, human eye) is proportional to radiance



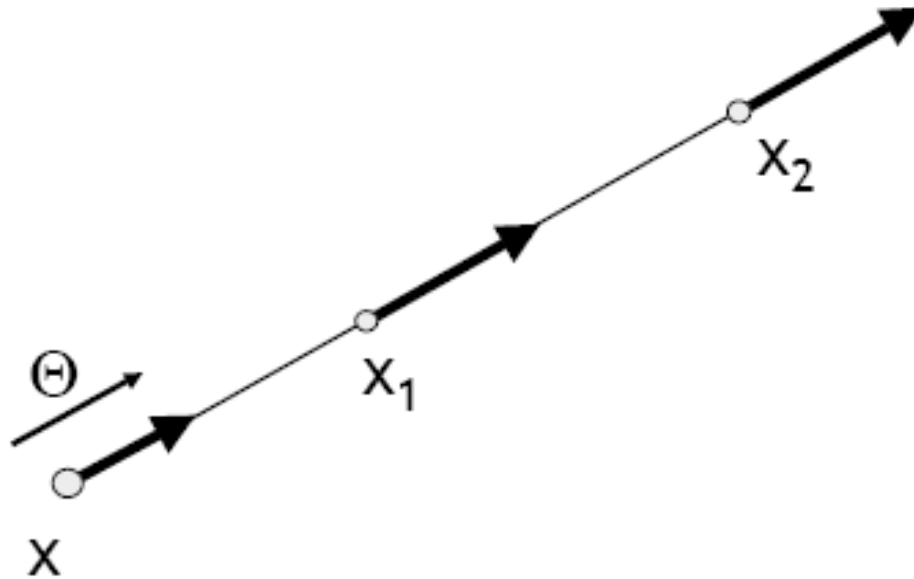
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- Pixel values in image proportional to radiance received from that direction

# Properties of Radiance

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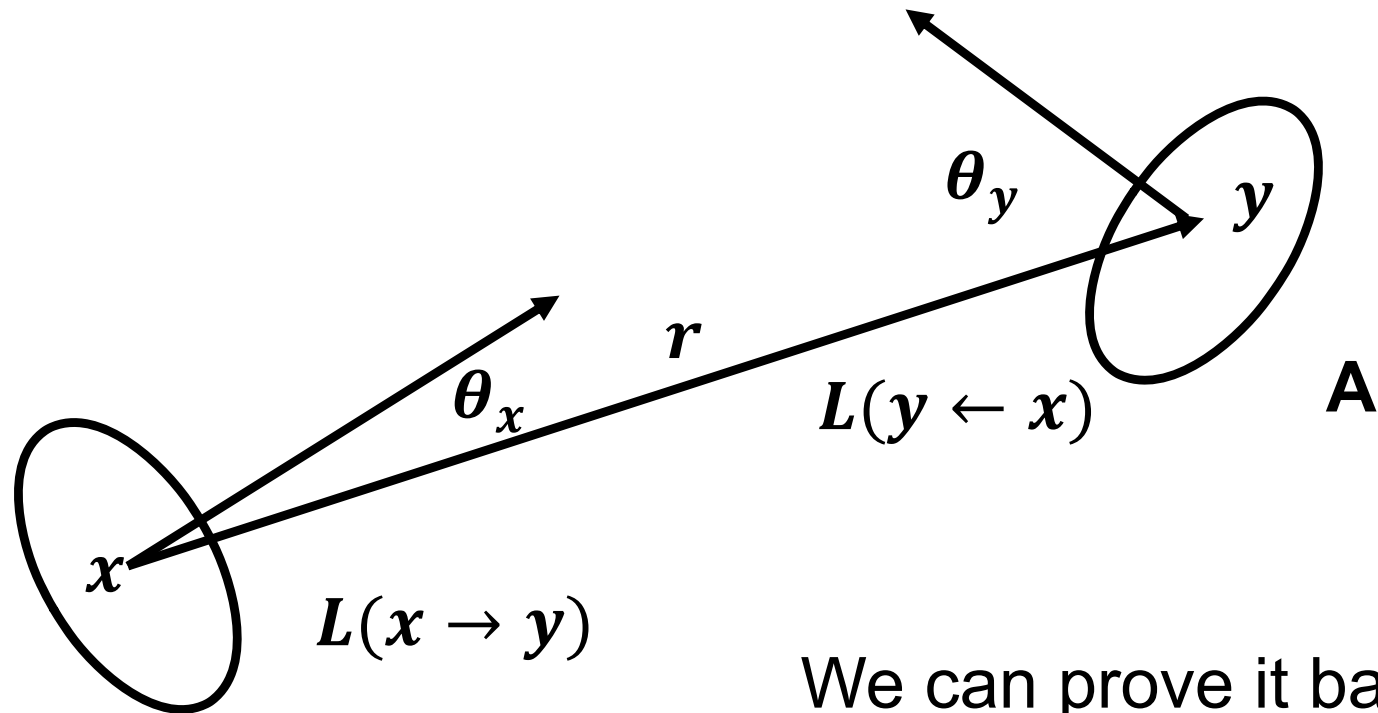
- **Invariant along a straight line (in vacuum)**



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# Invariance of Radiance

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We can prove it based on the assumption the conservation of energy.

# Relationships

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- Radiance is the fundamental quantity

$$L(x \rightarrow \Theta) = \frac{d^2P}{dA^\perp d\omega_\Theta}$$

- Power:

$$P = \int_{\text{Area}} \int_{\text{Solid Angle}} L(x \rightarrow \Theta) \cdot \cos \theta \cdot d\omega_\Theta \cdot dA$$

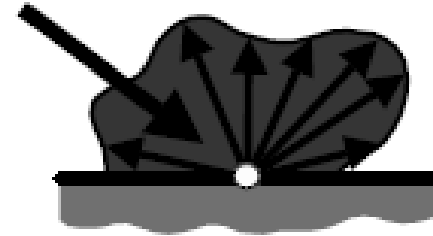
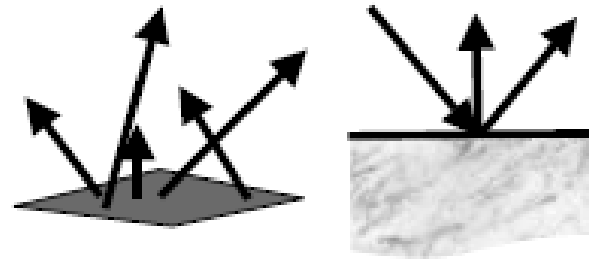
- Radiosity:

$$B = \int_{\text{Solid Angle}} L(x \rightarrow \Theta) \cdot \cos \theta \cdot d\omega_\Theta$$

# Light and Material Interactions

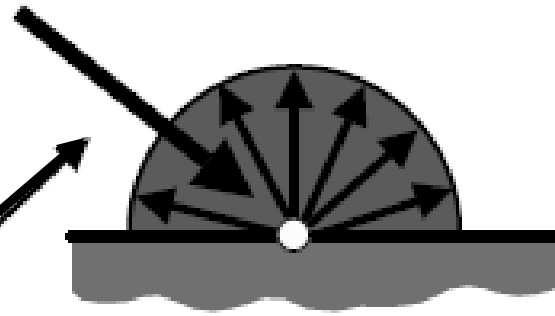
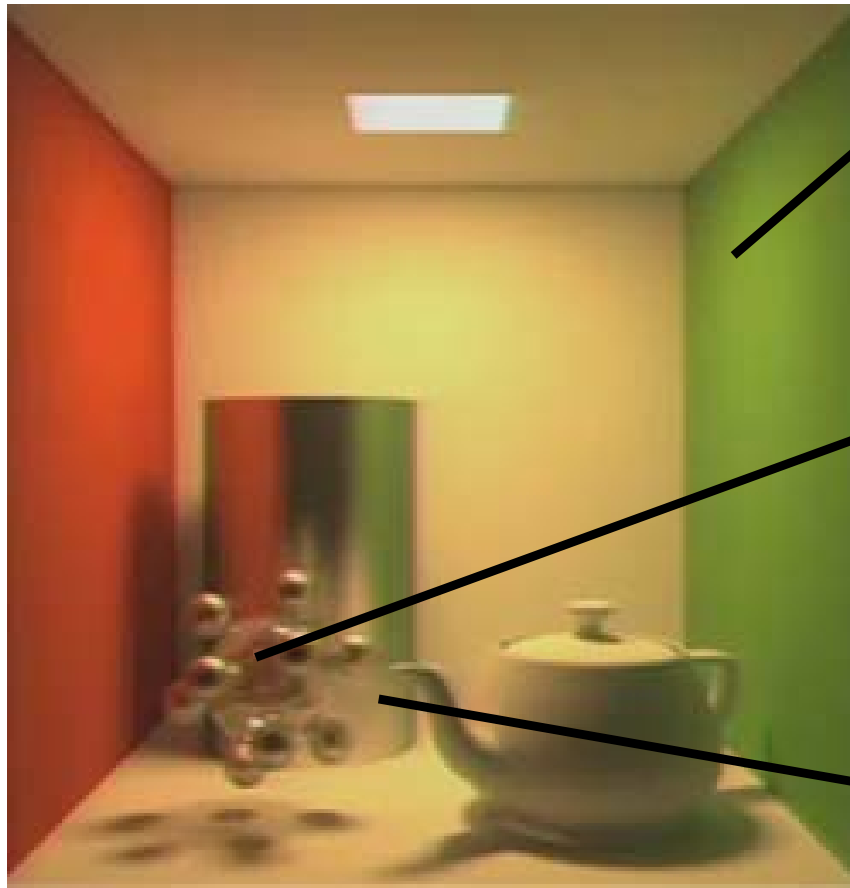
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- Physics of light
- Radiometry
- **Material properties**
- Rendering equation



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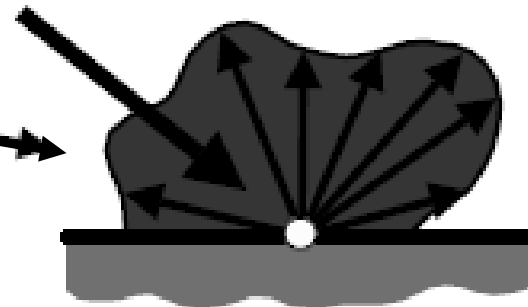
# Materials



**Ideal diffuse  
(Lambertian)**



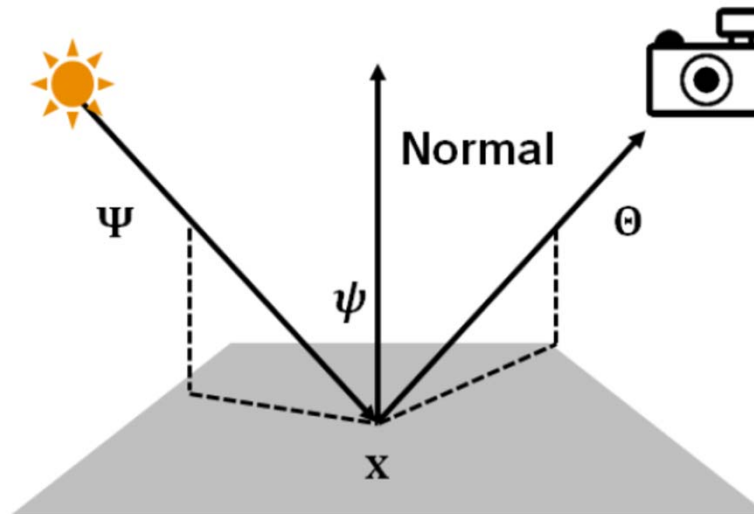
**Ideal specular**



**Glossy**

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# Bidirectional Reflectance Distribution Function (BRDF)



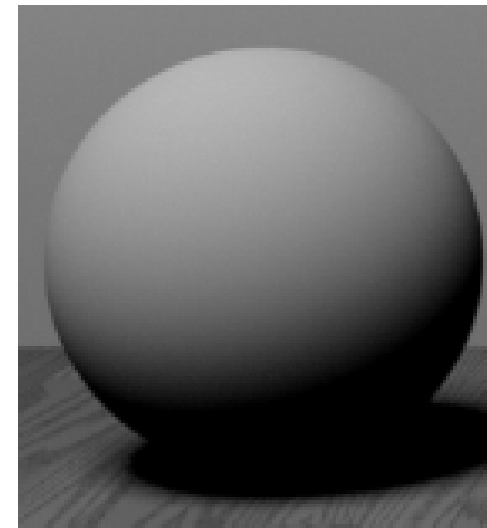
$$f_r(x, \Psi \rightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{dE(x \leftarrow \Psi)} = \frac{dL(x \rightarrow \Theta)}{L(x \leftarrow \Psi) \cos \psi d\omega_\Psi}$$

# BRDF special case: ideal diffuse

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## Pure Lambertian

$$f_r(x, \Psi \rightarrow \Theta) = \frac{\rho_d}{\pi}$$

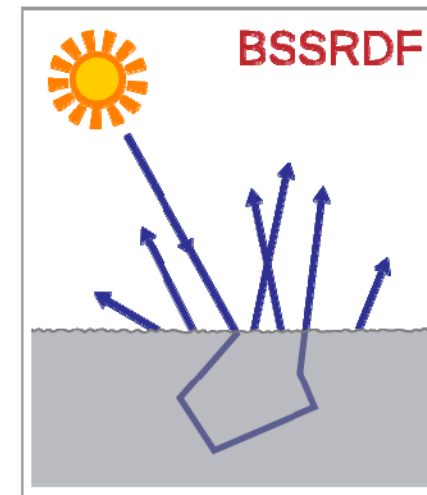
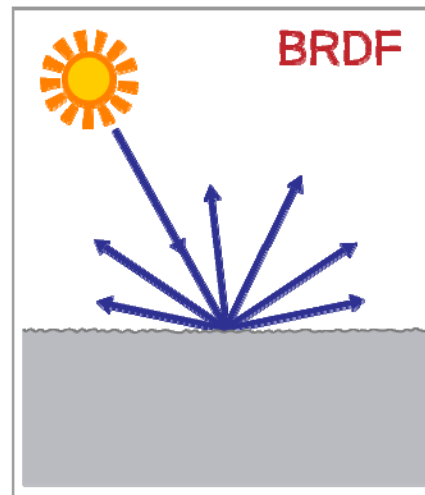
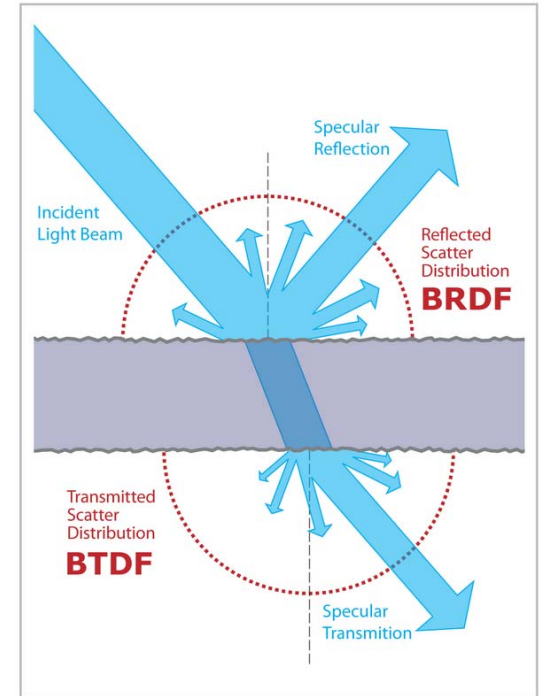


$$\rho_d = \frac{\text{Energy}_{out}}{\text{Energy}_{in}} \quad 0 \leq \rho_d \leq 1$$



# Other Distribution Functions: BxDF

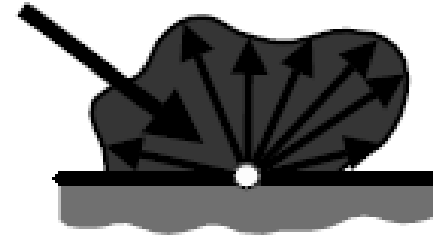
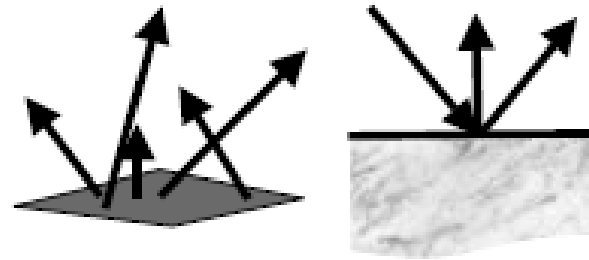
- **BSDF (S: Scattering)**
  - The general form combining **BRDF + BTDF (T: Transmittance)**
- **BSSRDF (SS: Surface Scattering)**
  - Handle subsurface scattering



# Light and Material Interactions

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- **Physics of light**
- **Radiometry**
- **Material properties**
- **Rendering equation**



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# Light Transport

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- **Goal**
  - **Describe steady-state radiance distribution in the scene**
- **Assumptions**
  - **Geometric optics**
  - **Achieves steady state instantaneously**

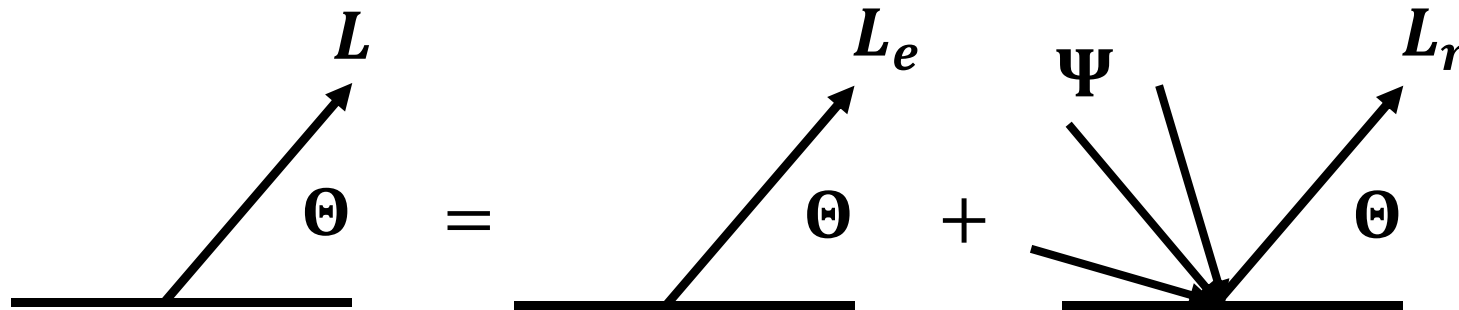
# Rendering Equation

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- **Describes energy transport in the scene**
- **Input**
  - **Light sources**
  - **Surface geometry**
  - **Reflectance characteristics of surfaces**
- **Output**
  - **Value of radiances at all surface points in all directions**

# Rendering Equation

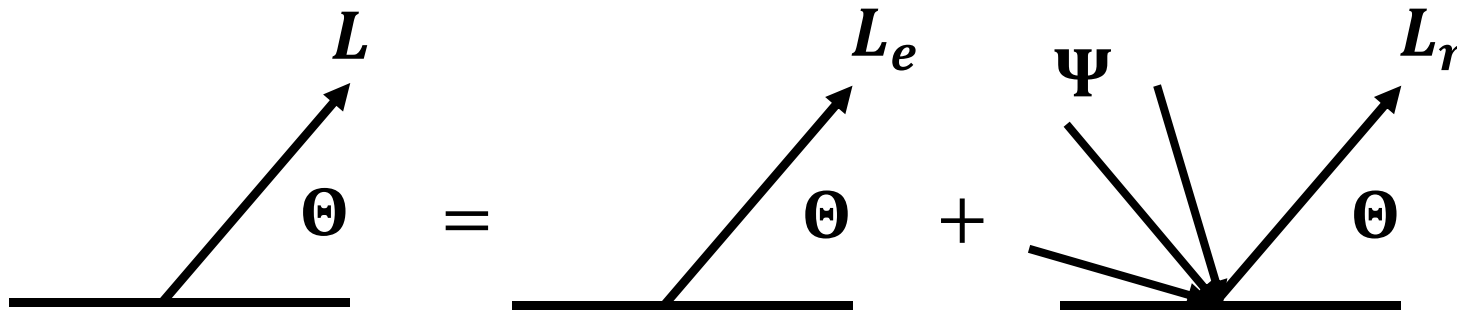
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$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + L_r(x \rightarrow \Theta)$$

# Rendering Equation

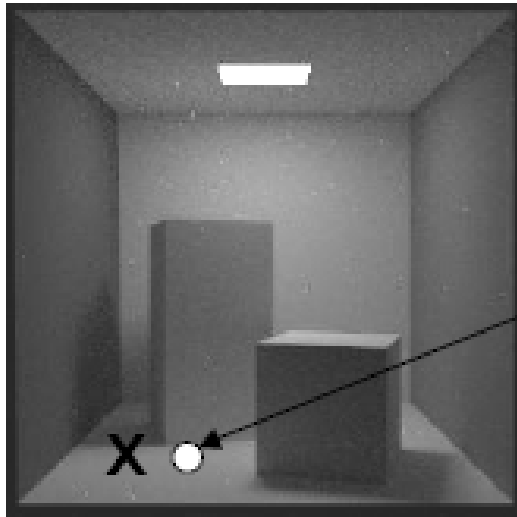
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$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos \theta_x d\omega_{\Psi},$$

- Applicable to all wave lengths

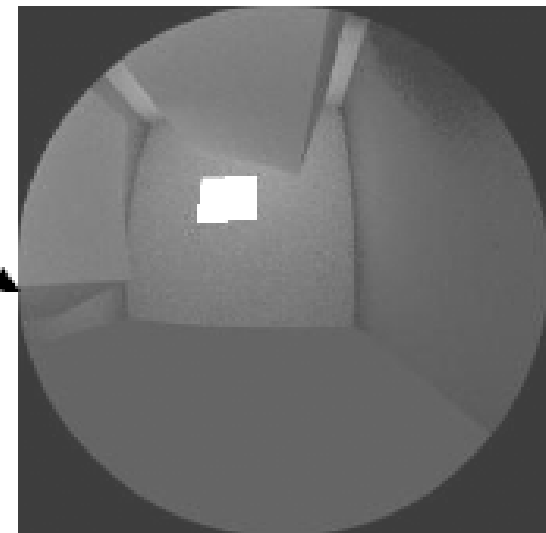
# Rendering Equation



$$\underline{L(x \rightarrow \Theta)} = L_e(x \rightarrow \Theta) +$$

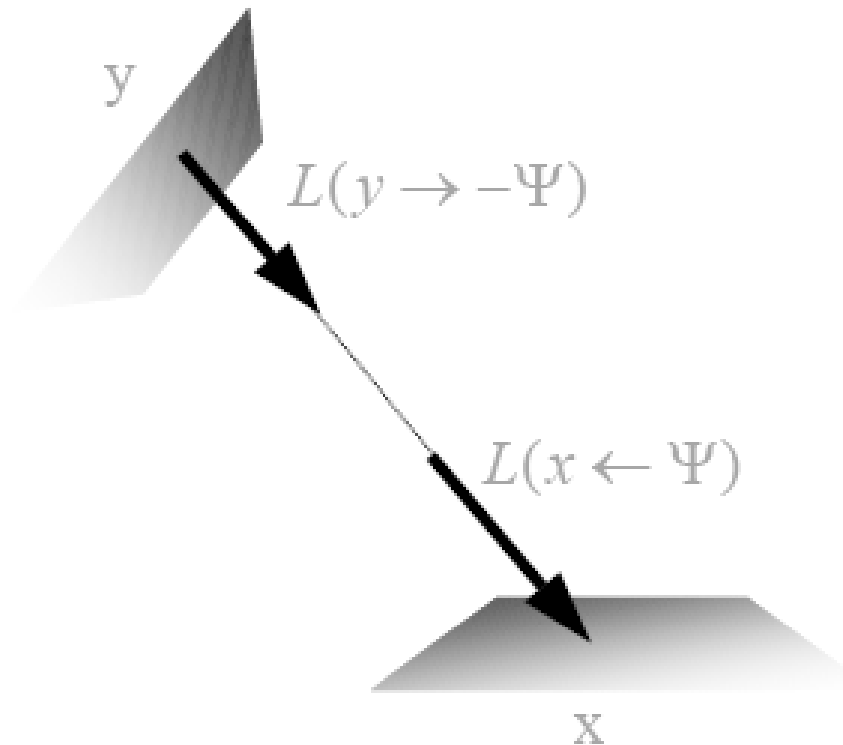
$$\int_{\text{hemisphere}} L(x \leftarrow \Psi) f_r(x, \Psi \leftrightarrow \Theta) \cos(\mathbf{N}_x, \Psi) d\omega_\Psi$$

incoming radiance



# Rendering Equation: Area Formulation

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos \theta_x \cdot d\omega_\Psi$$



Ray-casting function: what is the nearest visible surface point seen from  $x$  in direction  $\Psi$ ?

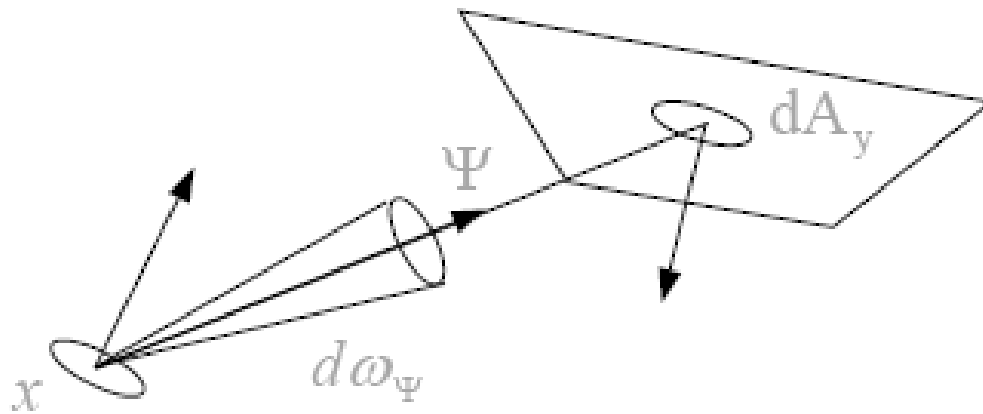
$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \rightarrow -\Psi)$$



# Rendering Equation

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos \theta_x \cdot d\omega_\Psi$$



$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \rightarrow -\Psi)$$

$$d\omega_\Psi = \frac{dA_y \cos \theta_y}{r_{xy}^2}$$

# Rendering Equation: Visible Surfaces

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos \theta_x \cdot d\omega_\Psi$$

Coordinate transform



$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\substack{y \text{ on} \\ \text{all surfaces}}} f_r(\Psi \leftrightarrow \Theta) \cdot L(y \rightarrow -\Psi) \cos \theta_x \cdot \frac{\cos \theta_y}{r_{xy}^2} \cdot dA_y$$



$$y = vp(x, \Psi)$$

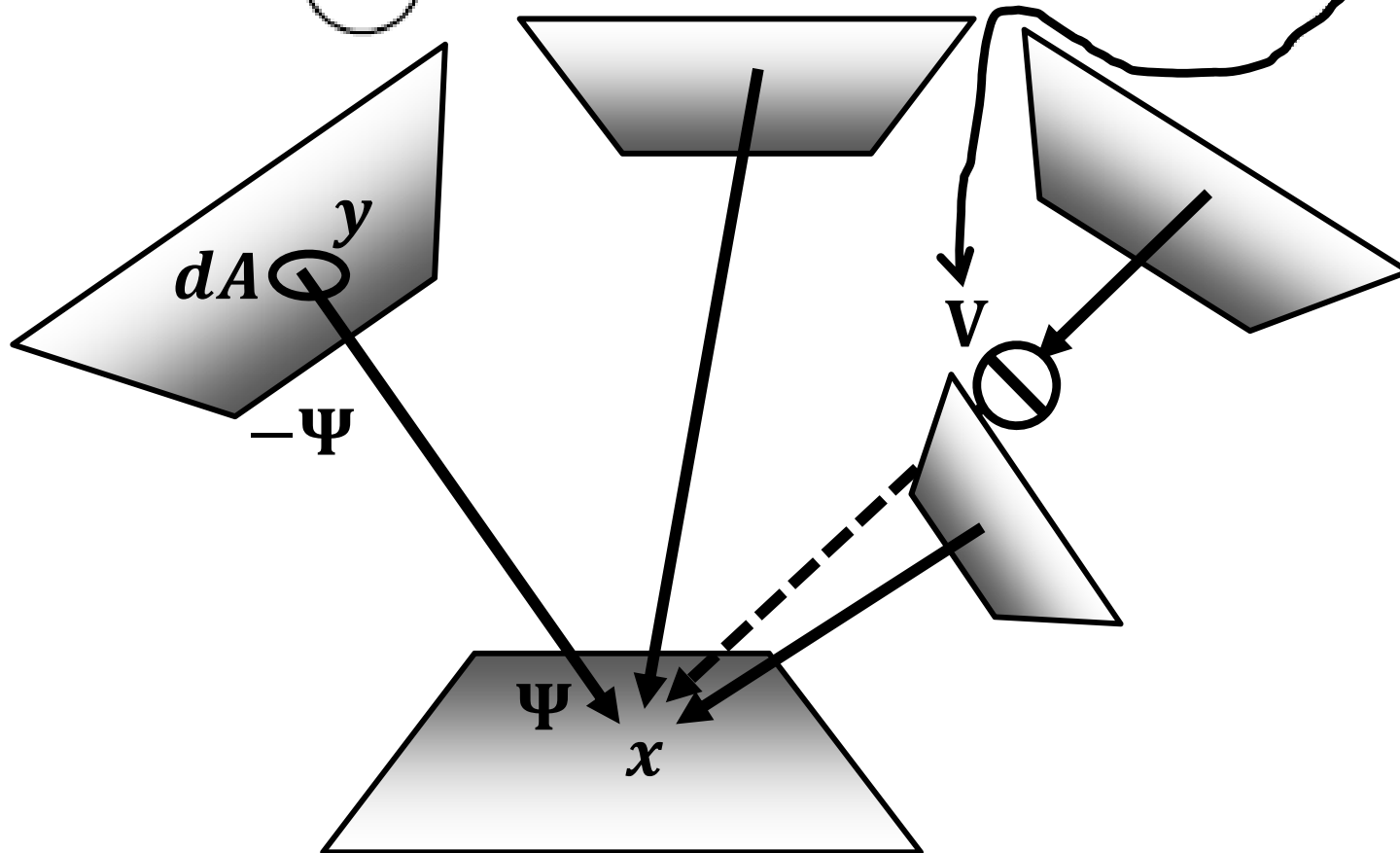


Integration domain = visible surface points  $y$

- Integration domain extended to ALL surface points by including visibility function

# Rendering Equation: All Surfaces

$$L(x \rightarrow \Theta) = L_e(\dots) + \int_A f_r(\dots) \cdot L(y \rightarrow -\Psi) \cdot \frac{\cos \theta_x \cdot \cos \theta_y}{r_{xy}^2} \cdot V(x, y) dA_y$$



# Two Forms of the Rendering Equation

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- **Hemisphere integration**

$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos \theta_x d\omega_{\Psi}$$

- **Area integration (used as the form factor)**

$$L_r(x \rightarrow \Theta) = \int_A L(y \rightarrow -\Psi) f_r(x, \Psi \rightarrow \Theta) \frac{\cos \theta_x \cos \theta_y}{r_{xy}^2} V(x, y) dA,$$

# **Class Objectives (Ch. 12 & 13) were:**

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- **Know terms of:**
  - **Hemispherical coordinates and integration**
  - **Various radiometric quantities (e.g., radiance)**
  - **Basic material function, BRDF**
  - **Understand the rendering equation**

# Next Time

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- **Monte Carlo rendering methods**

# Homework

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- **Go over the next lecture slides before the class**
- **Watch two videos or go over papers, and submit your summaries every Tue. class**
  - **Just one paragraph for each summary**

## Example:

**Title: XXX XXXX XXXX**

**Abstract: this video is about accelerating the performance of ray tracing. To achieve its goal, they design a new technique for reordering rays, since by doing so, they can improve the ray coherence and thus improve the overall performance.**

# Any Questions?

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- **Submit three times before the mid-term exam**
- **Come up with one question on what we have discussed in the class and submit at the end of the class**
  - **1 for already answered questions**
  - **2 for questions that have some thoughts or surprise me**





# Fig: Invariance of Radiance

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