Real-time Continuous Collision Detection and Penetration Depth Computation

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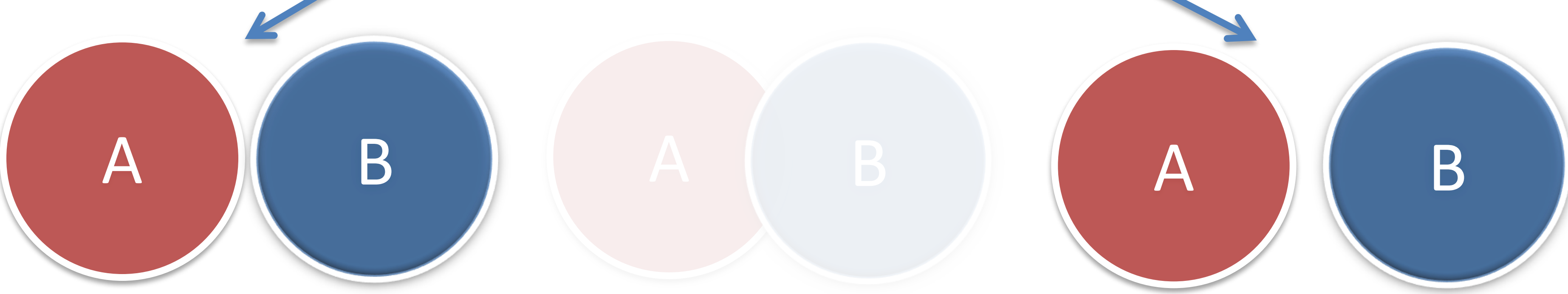
Non-penetration Constraint

- No overlap between geometric objects

$$\text{interior}(A) \cap B \neq \phi$$



- Crucial for mimicking the physical presence

Earlier Research

- Focused on checking for whether there is any overlap between A and B, *fixed in space*
  - Tons of papers published in the area of *collision detection*
  - Well-studied and matured technology
- Not clear how to resolve such overlap

Recent Research Trends

- Avoid inter-penetration
  - Continuous collision detection (CCD)
- Allow inter-penetration but backtrack
  - Penetration depth (PD)

Autonomous Ice Serving Robot
(with Zhixing Xue at FZI)

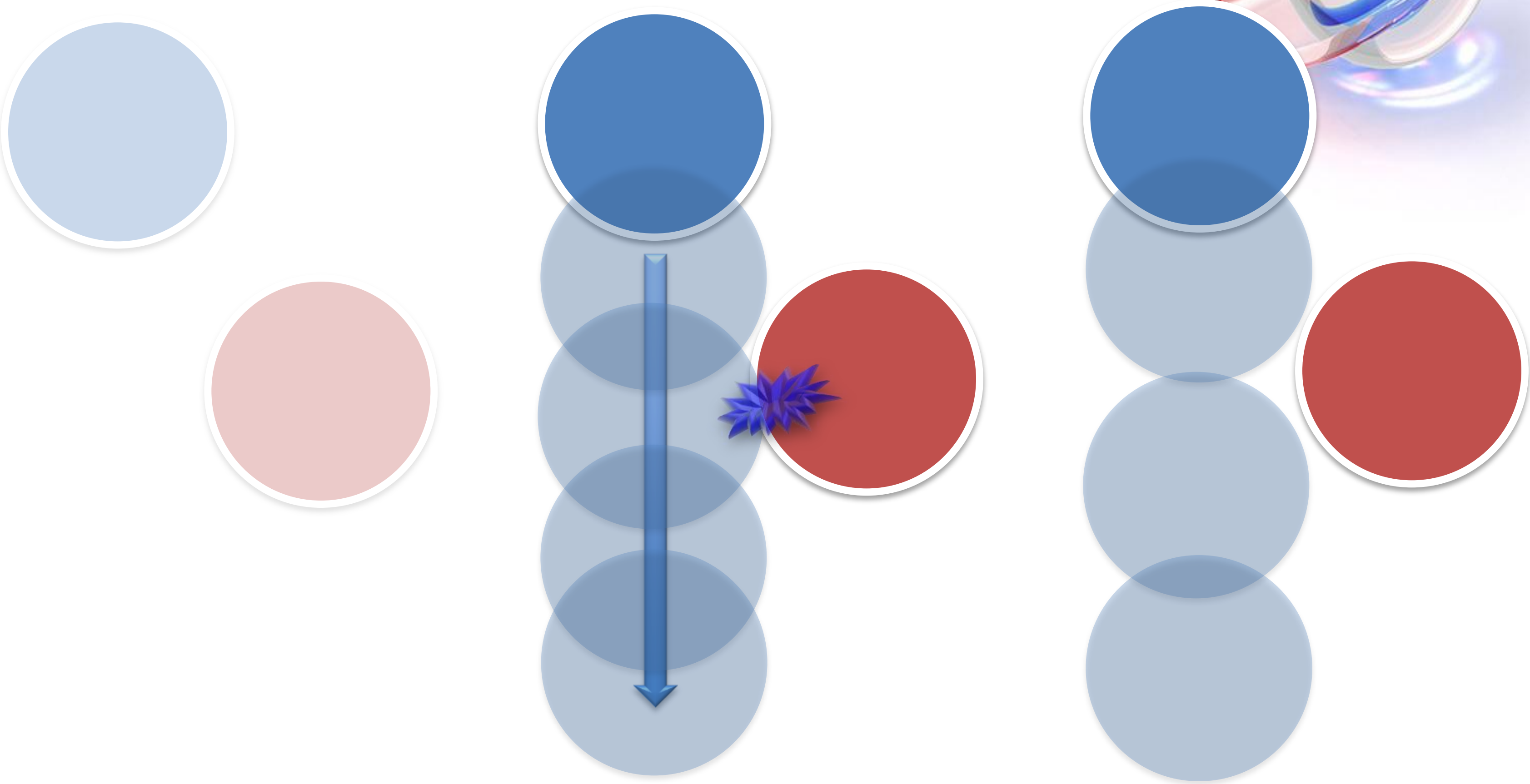
Outline

- Recent research results
  - CCD (rigid, polygon-soups, articulated)
  - PD (translational)

- Applications
  - Real-time rigid body dynamics
  - Robotic grasping
  - CAD disassembly

CONTINUOUS COLLISION DETECTION (CCD)

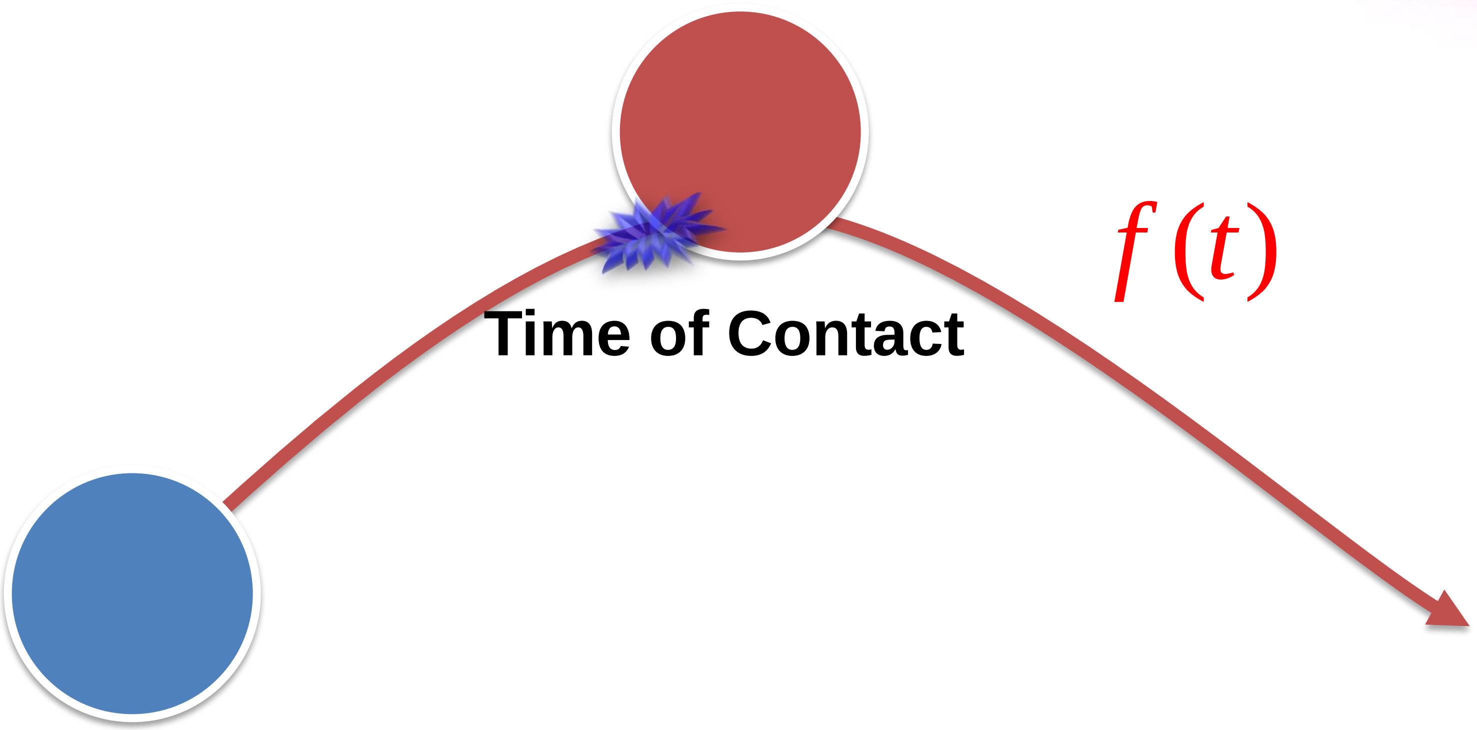
Discrete Collision Detection



Collision Missing

Continuous Collision Detection

- Motion trajectory ***f(t)*** is known in advance

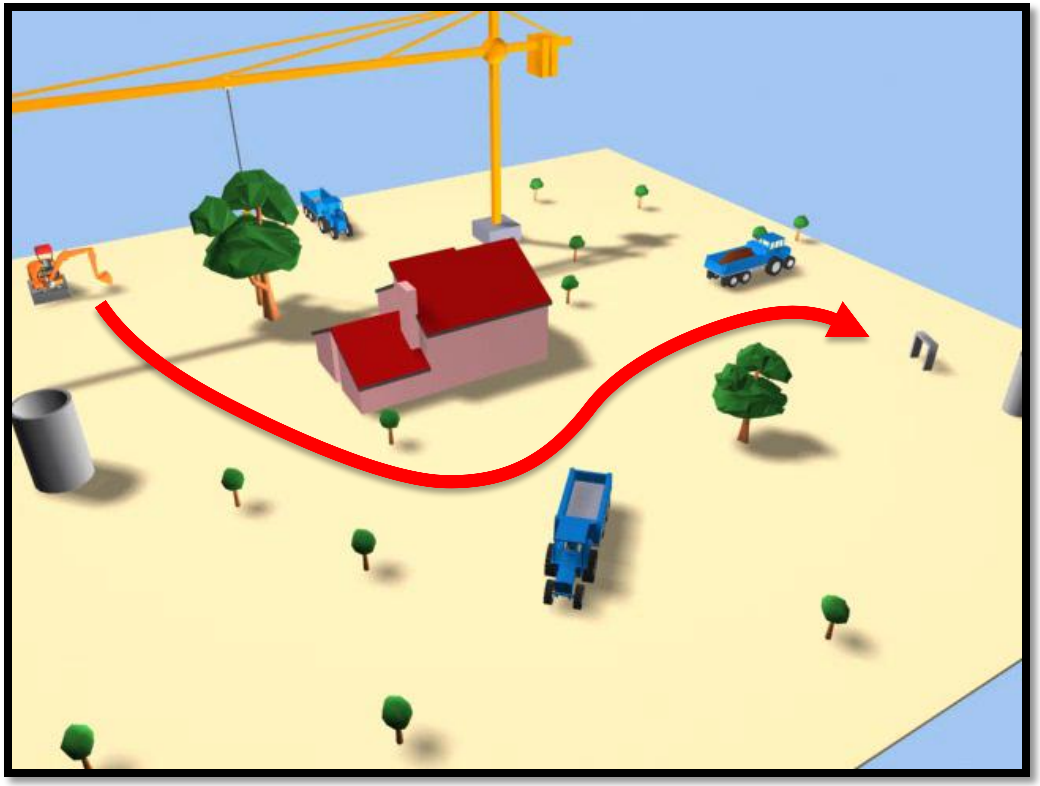


Applications of Continuous CD

- Rigid body dynamics
  - Find the time of contact (ToC) to apply forces

Applications of Continuous CD

- Motion planning
  - Check whether a path is collision-free

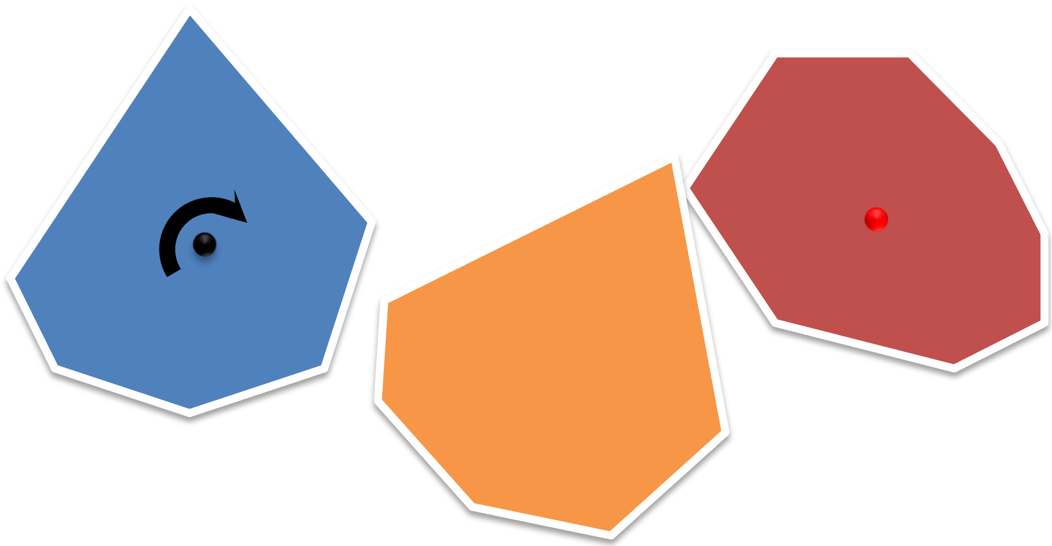


Previous Work on CCD

- Algebraic solution -[Canny86], [Redon00], [Kim03], [Choi06]
- Swept volume -[Abdel-Malek02],[Hubbard93], [Redon04a,b]
- Bisection -[Redon02], [Schwarzer02]
- Kinetic data structures -[Kim98], [Kirkpatrick00], [Agarwal01]
- Minkowski sum -[Bergen04]
- Conservative advancement
  - [Mirtich96], [Mirtich00],[Coumans 2006],[Zhang06], [Tang09]

Conservative Advancement (CA)

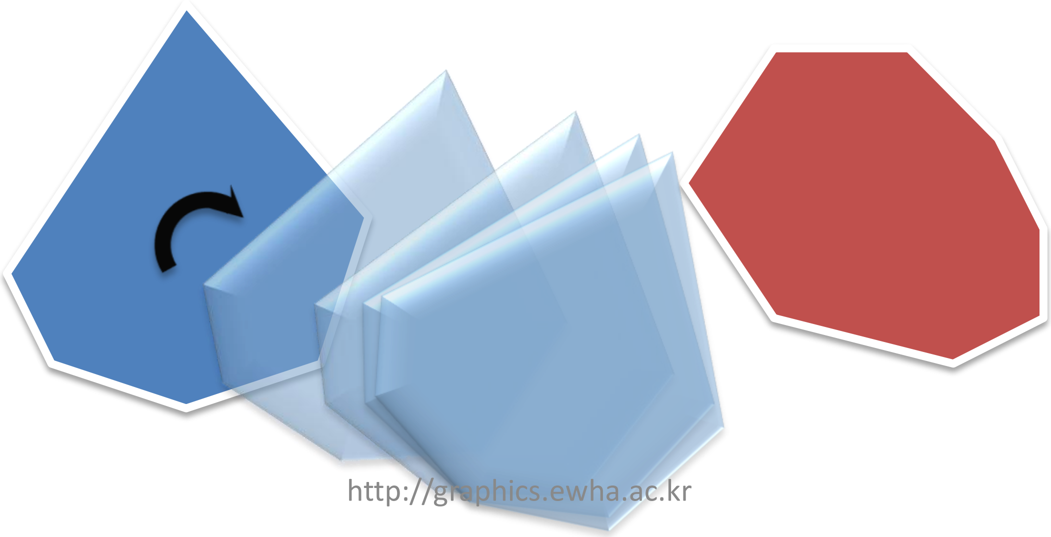
- Assume objects are *convex*
- Find the 1st time of contact (ToC) of a moving object



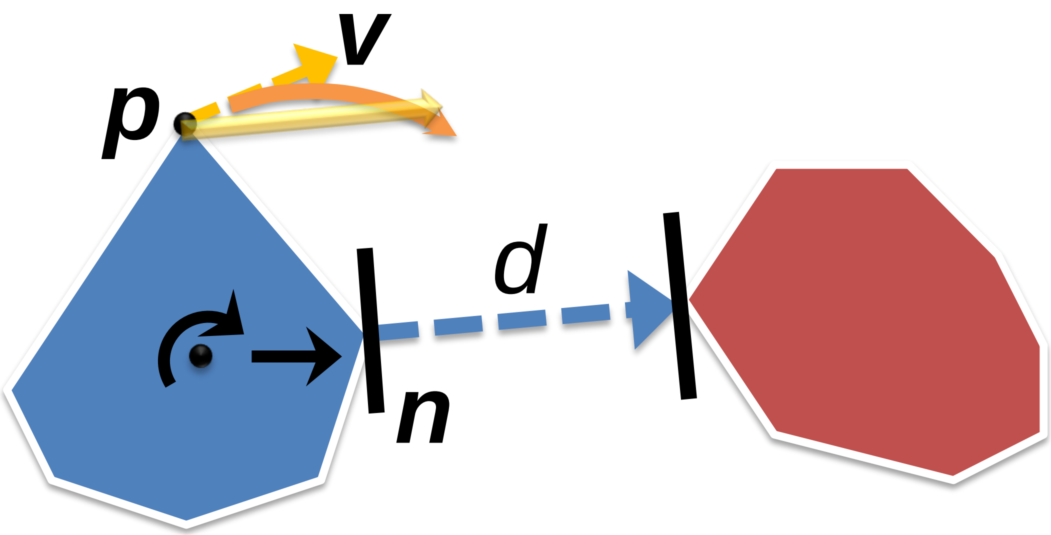
Conservative Advancement (CA)

1. Find a step size Δt_i to conservatively advance the object before collision occurs
2. Repeat until inter-distance < ε

$$ToC = \Delta t_1 + \Delta t_2 + \Delta t_3 + \Delta t_4$$



Calculating Δt in CA



d, ***n***: closest distance, direction vector

v: velocity

$$\int_0^{\Delta t} |v(t)\bullet n(t)|\,dt \le \int_0^{\Delta t} \boxed{\max(|v(t)\bullet n(t)|)}\,dt = \mu$$

Calculating Δt in CA

$$\mu \Delta t \le d \qquad \therefore \Delta t \le \frac{d}{\mu}$$

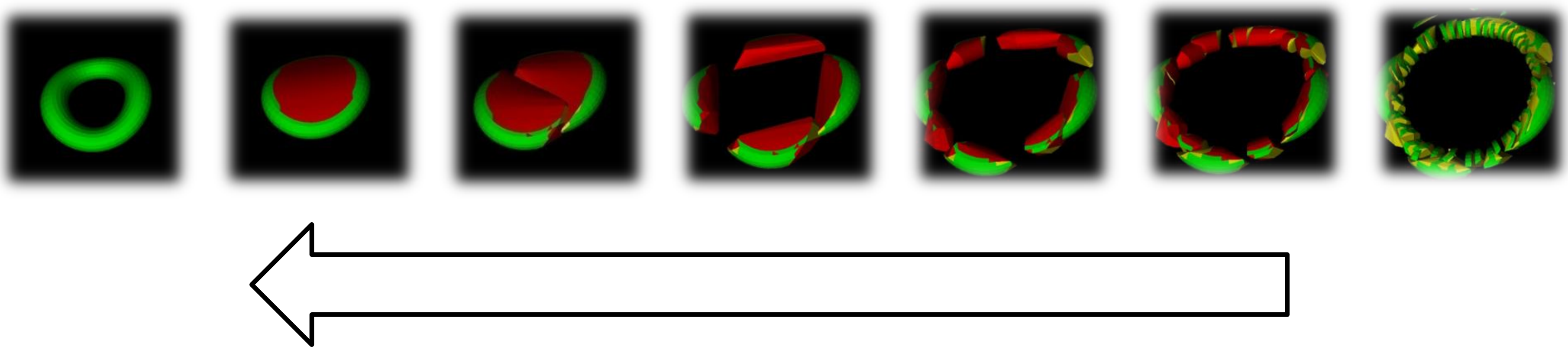
$$\int_0^{\Delta t} |v(t) \bullet n(t)|\, dt \le \int_0^{\Delta t} \overset{=\mu}{\boxed{\max(|v(t) \bullet n(t)|)}}$$

Distance

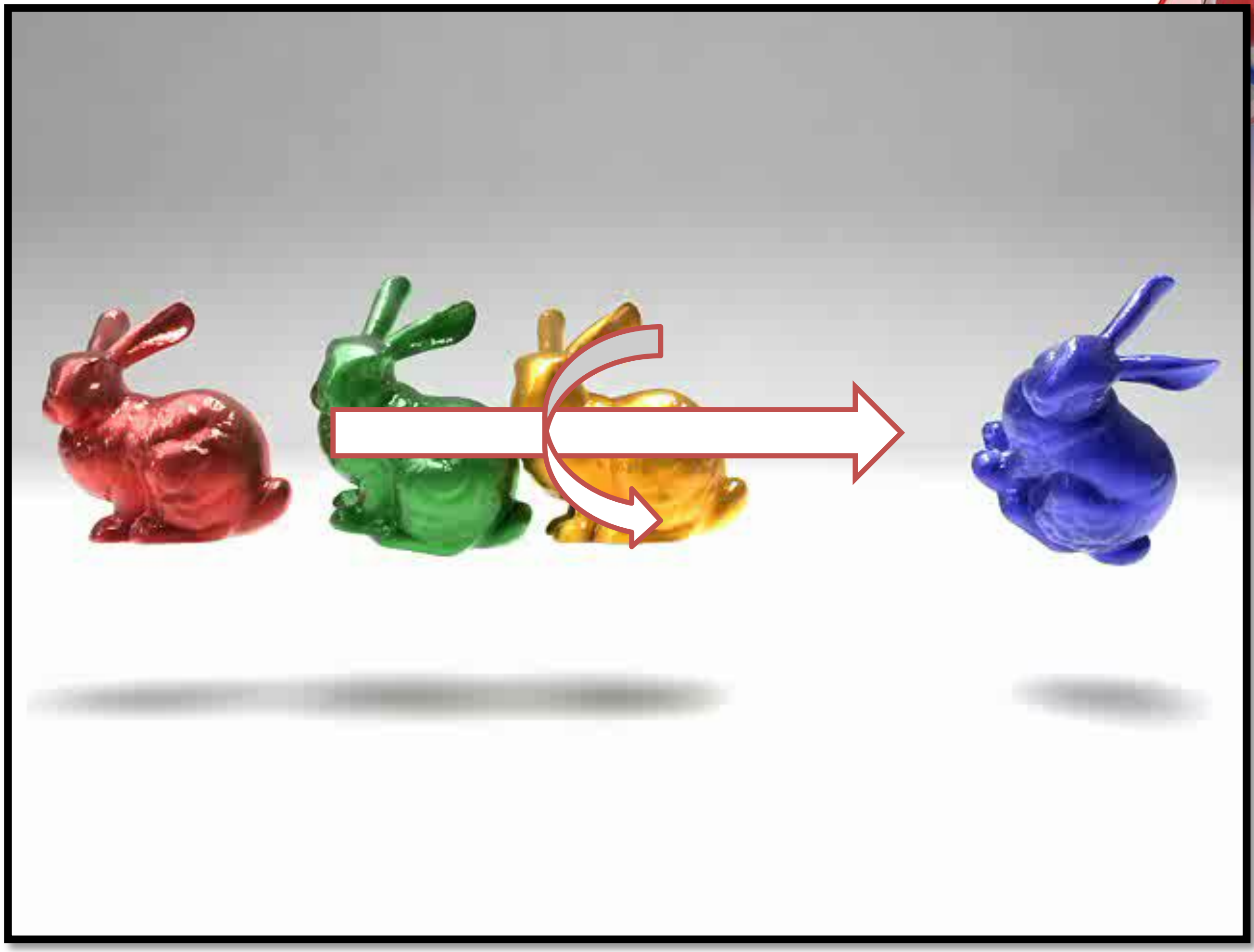
Motion Bound

Extension to Non-convex
[Zhang et. al PG 06]

- Use of convex decomposition
- Build a hierarchy of decomposed convex pieces and perform CA *hierarchically*

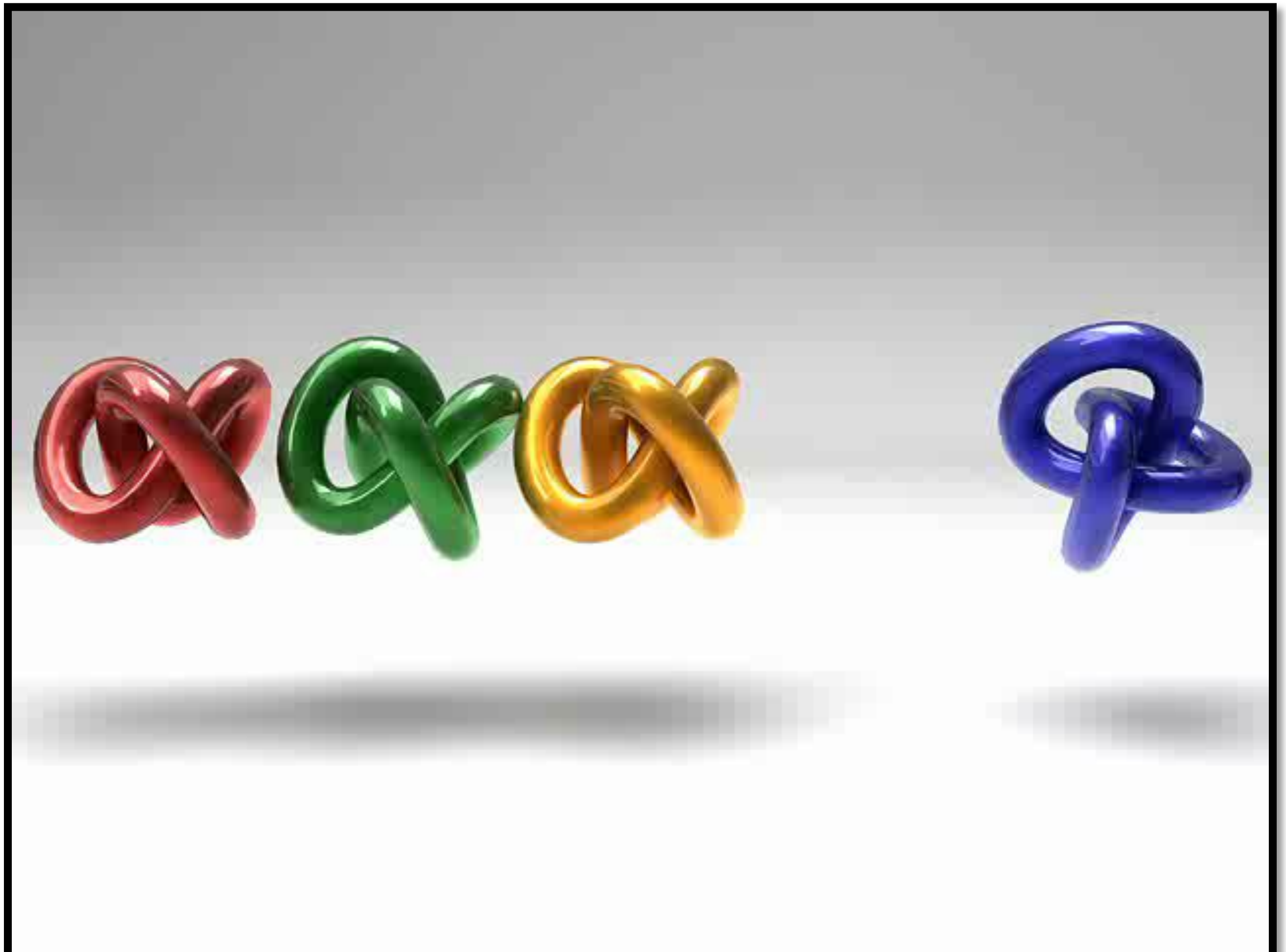


Bunny vs. Bunny



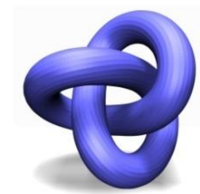
70K triangles/bunny 110 FPS # of iterations 4.7

Torusknot vs. Torusknot



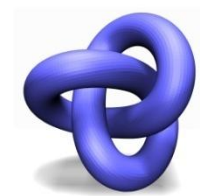
2.8K
1067 FPS

# of iterations 4.49



11K
400 FPS

# of iterations 4.49



34K
186 FPS

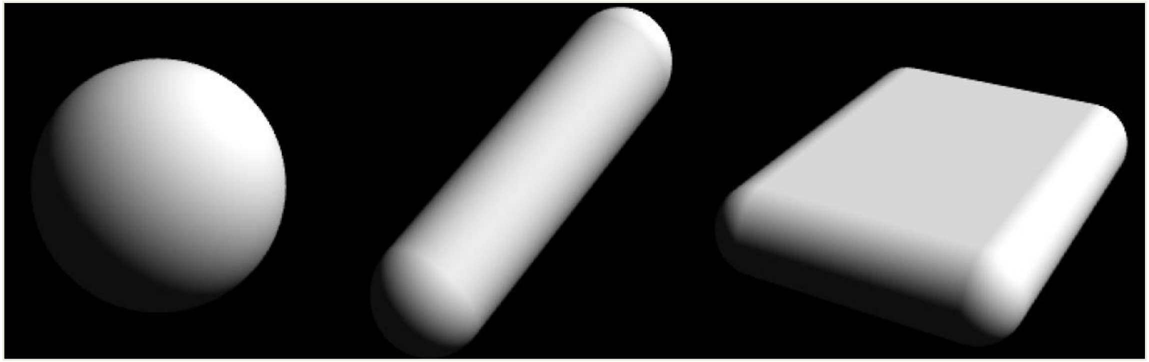
# of iterations 4.46

Extension to Polygon-Soups
[Tang et. al IEEE ICRA 09]

- Construct the bounding volume hierarchy of polygons
- Motion bound calculation $\Delta t \le \dfrac{d}{\mu}$
  - Bounding volume
  - Triangles

Extension to Polygon-Soups
[Tang et. al IEEE ICRA 09]

- We use swept sphere volumes



[Larsen et. al IEEE ICRA 1999]

Motion Bound Calculation

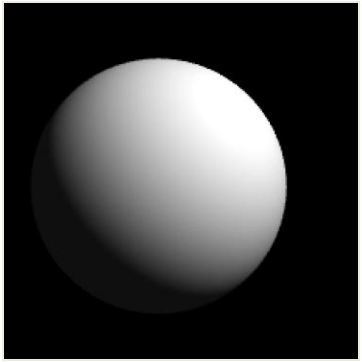
- Motion bound of SSV (e.g. PSS)

$$\mu \le \left\|\boldsymbol{\omega}\times\mathbf{n}\right\|\left(\left\|\mathbf{c}_1^{\perp}\right\|+\mathbf{r}\right)$$

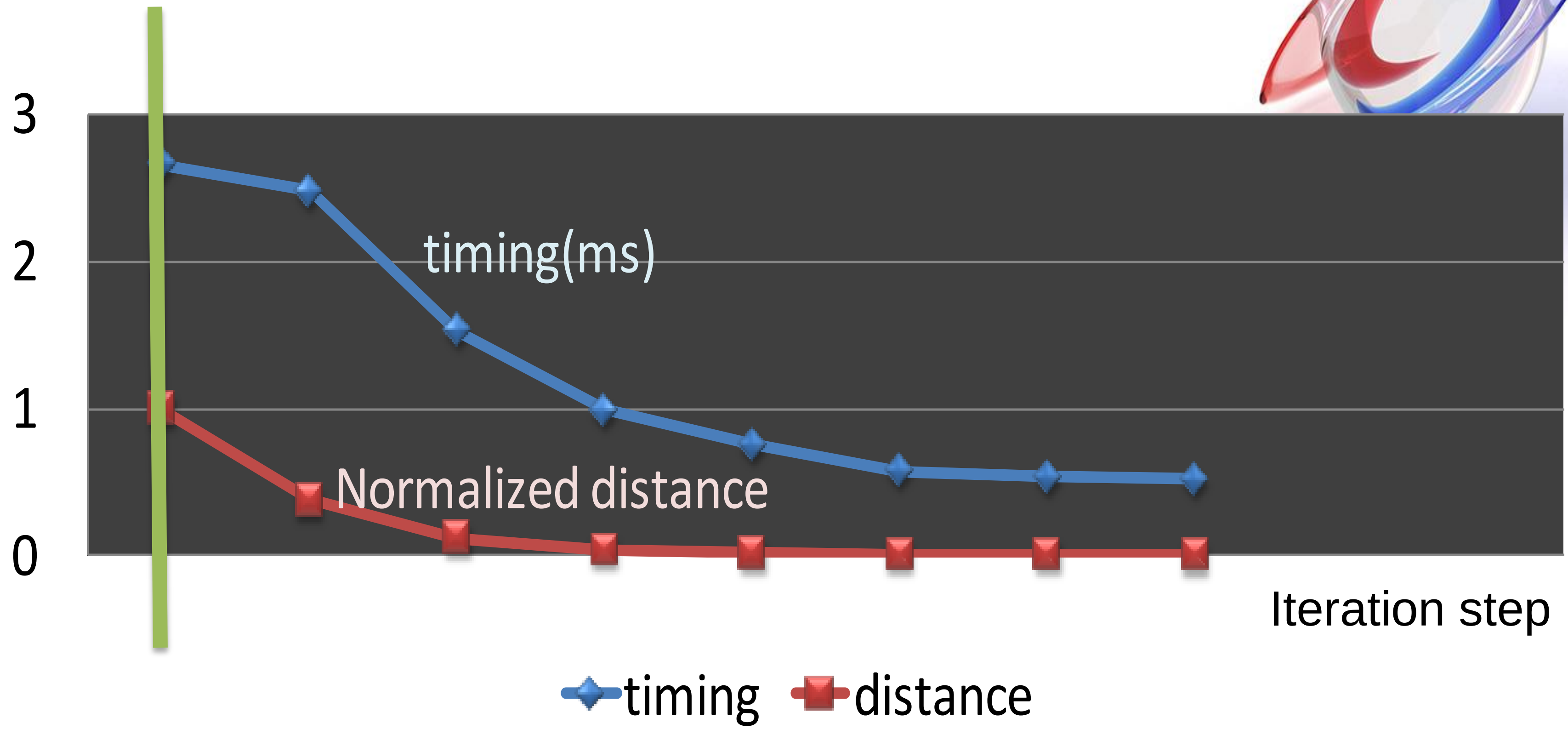
$\boldsymbol{\omega}$: rotational velocity

$\mathbf{n}$: closest direction

$\mathbf{r}$: radius of PSS

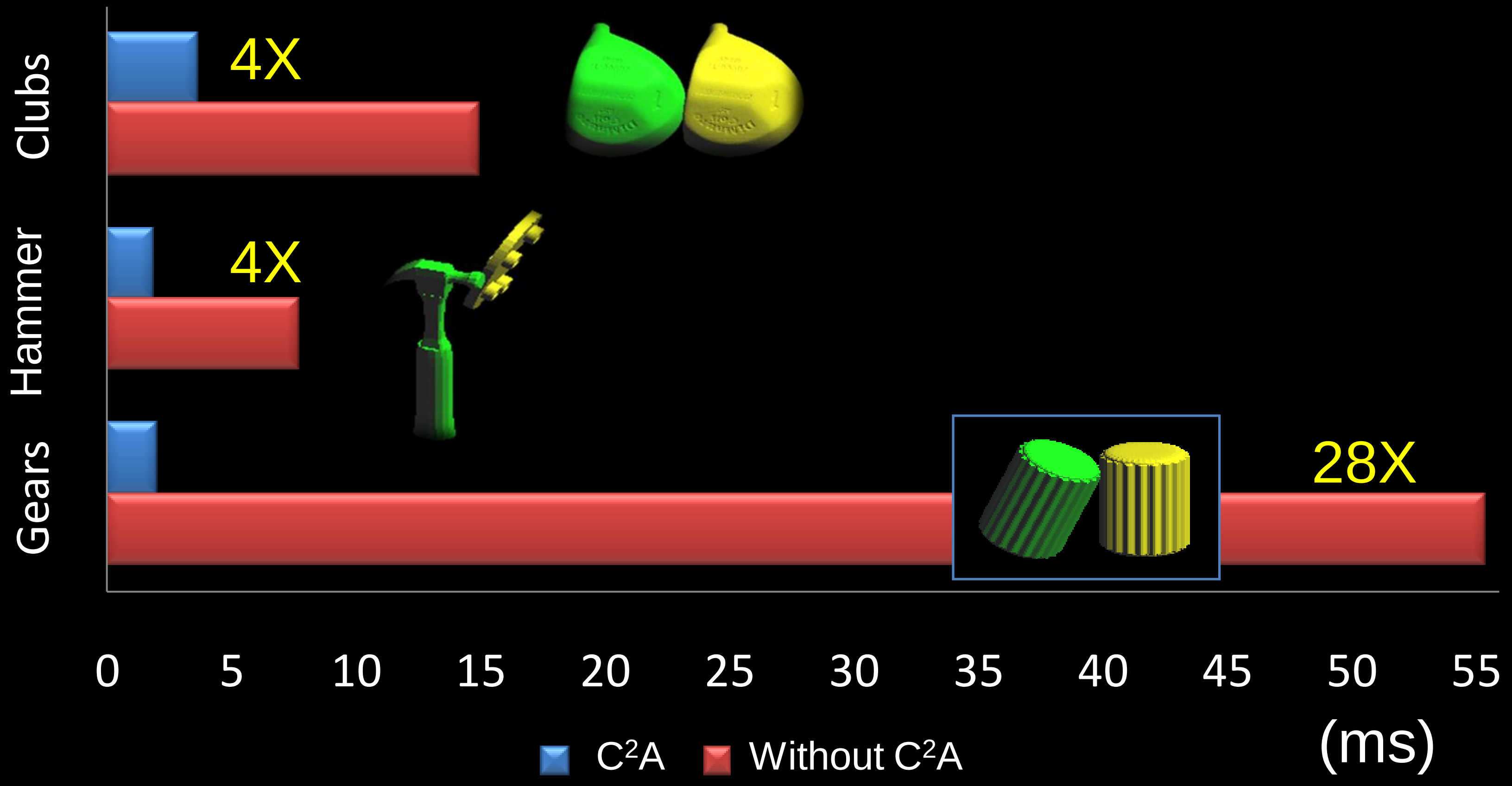


C²A:Controlling the CA Iterations

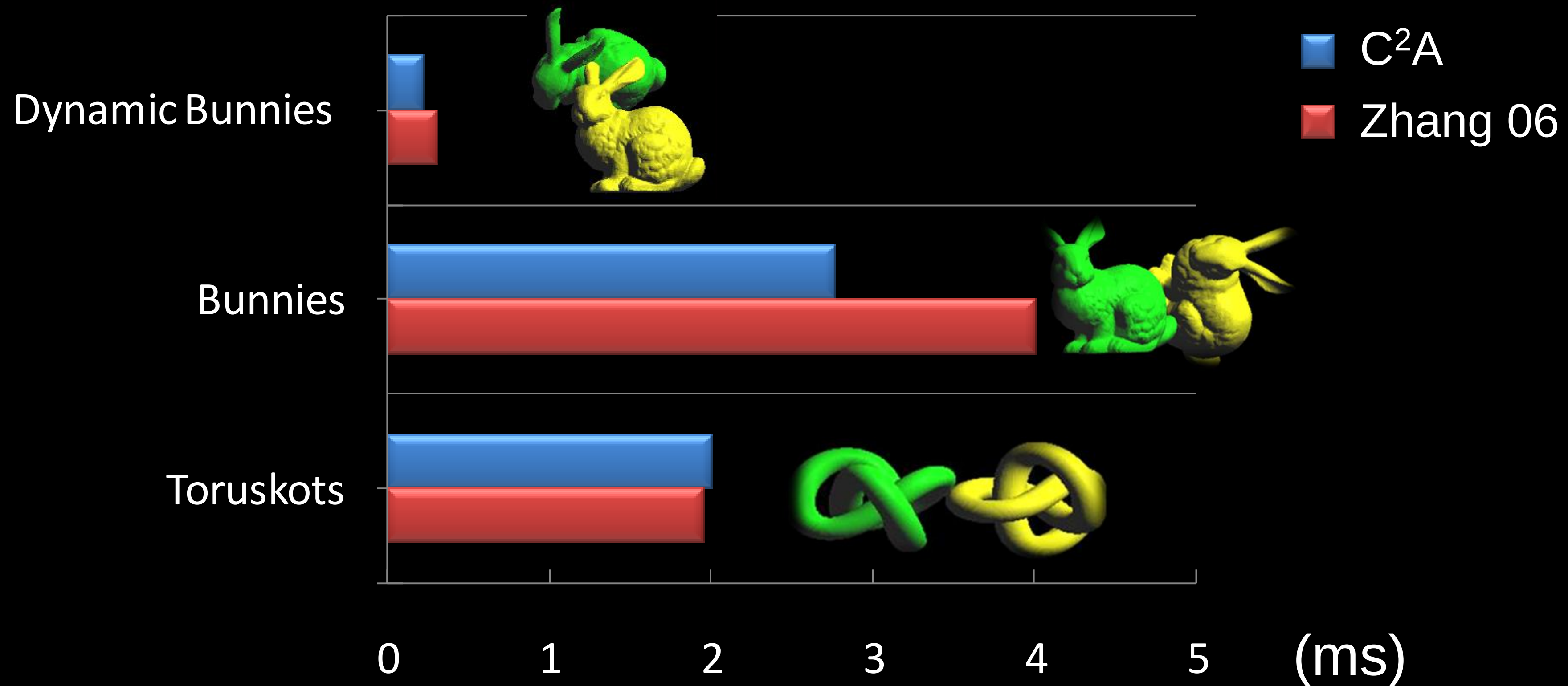


1. Compute approximate distance in the beginning
2. Compute exact distance toward the end

Results - Timings



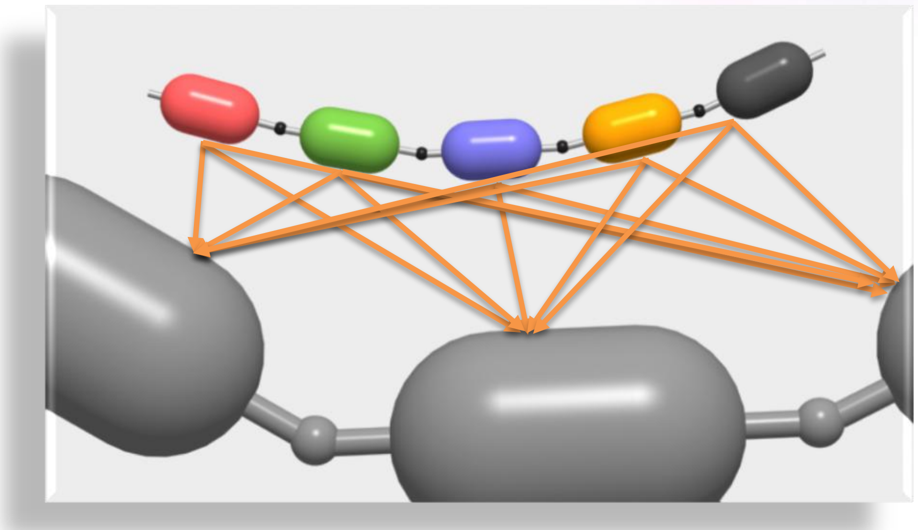
Comparisons against [Zhang 06]



- [Zhang 06] can handle only manifold surfaces

Extension to Articulated Models
[Zhang et. al SIGGRAPH 07]

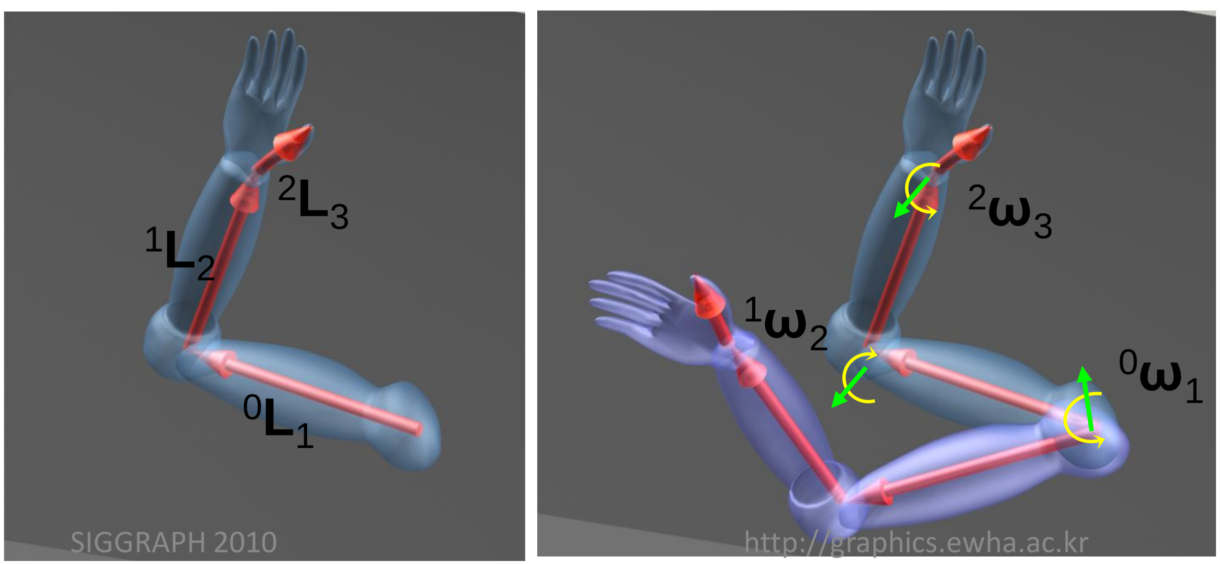
- Treat each link as a rigid body
- Apply CA to each link independently
- Taking the minimum of CA results



Motion Bound Calculation $\Delta t \le \dfrac{d}{\mu}$

μ

$$\le \left|{}^0\mathbf{v}_1\cdot\mathbf{n}\right| + \left|\mathbf{n}\times{}^0\boldsymbol{\omega}_1\right|\left|{}^0\mathbf{L}_1\right|_\mu + \sum_{j=2}^{i-1}\left(\left|{}^{j-1}\mathbf{v}_j\right| + \left(\left|\mathbf{n}\times{}^0\boldsymbol{\omega}_1\right| + \sum_{k=2}^{j}\left|{}^{k-1}\boldsymbol{\omega}_k\right|\right)\left|{}^{j-1}\mathbf{L}_j\right|_\mu\right)$$



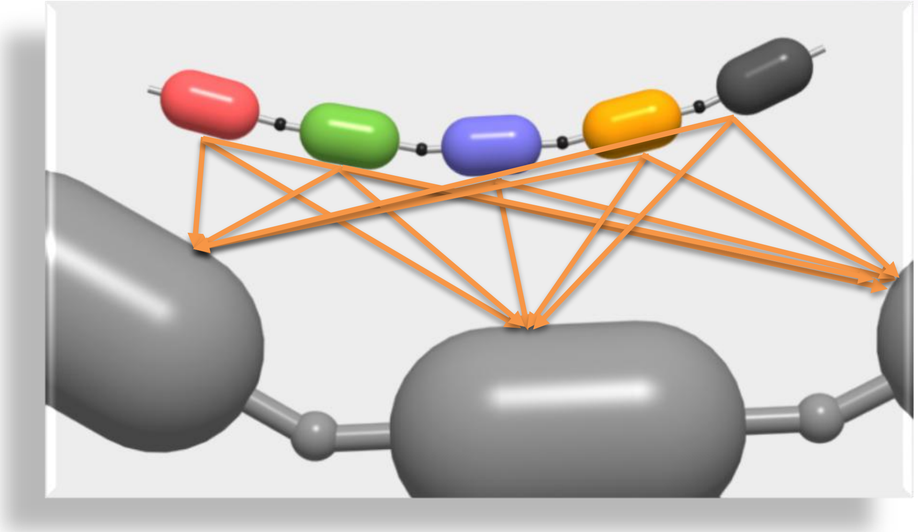
${}^0v_i$: velocity of link i

${}^{i-1}\omega_i$: rotational velocity of link i w.r.t. link i-1

${}^{i-1}L_i$: difference vector btwn the links

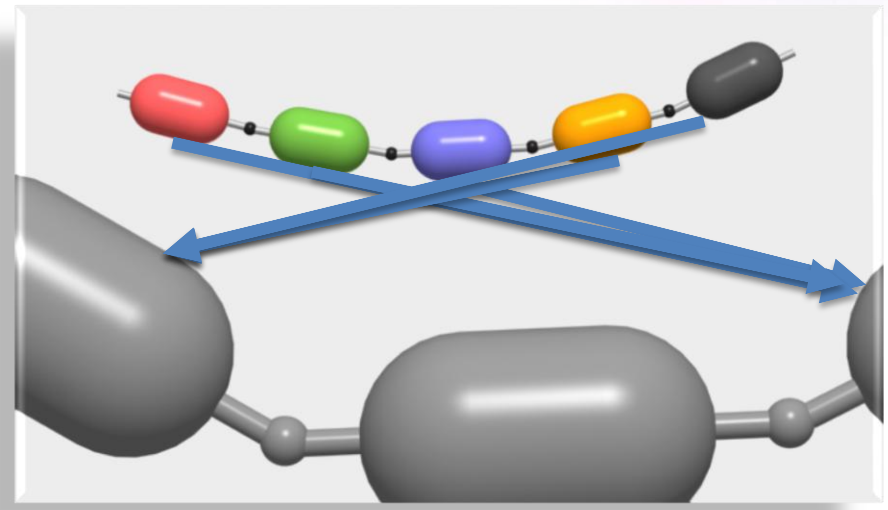
Problems in Straightforward CA

- Problems
  - O(n²) checking between individual links



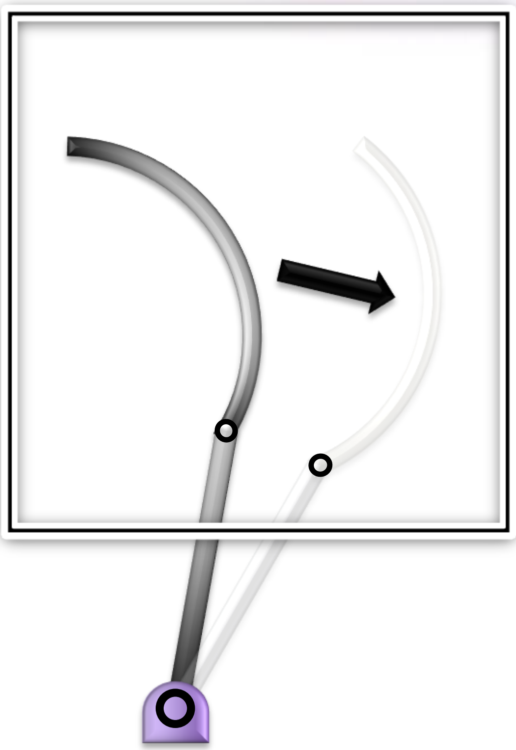
Spatial Culling

- Cull the link pairs that are far apart
- Use bounding volume-based collision-culling

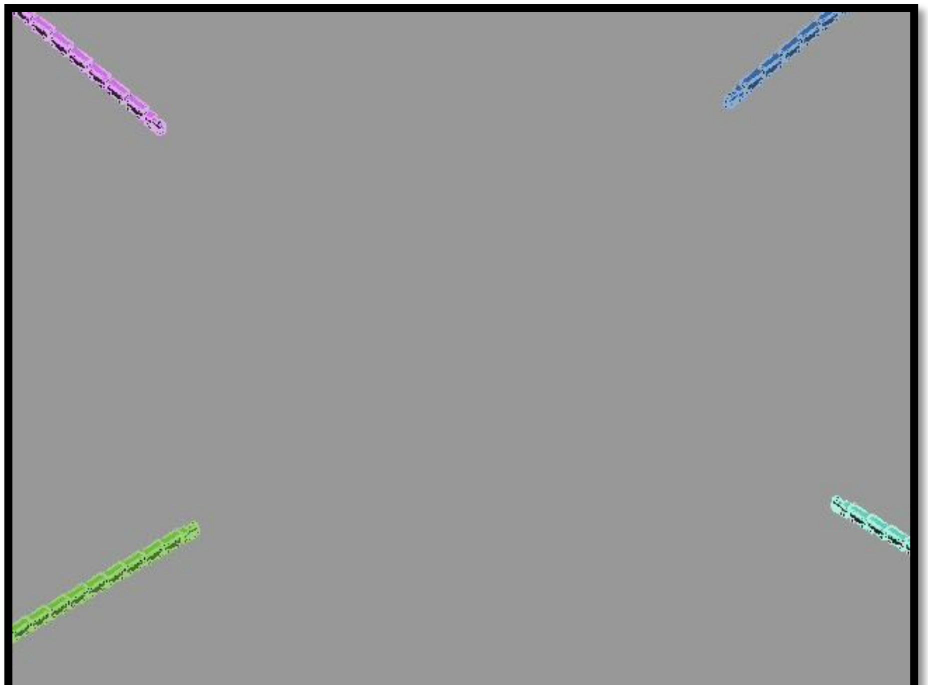


Spatial Culling using Dynamic AABB

- Goal
  - Compute an axis-aligned bounding box (AABB) that bounds the motion of a moving link



Bounding Volume Culling



Interval Arithmetic

Taylor Models

Locomotion Benchmark

- **CCD performance**
  - **1.22 msec**
- Mannequin
  - 15 links, 20K tri
- Obstacles
  - 101K tri
- Locomotion SW
  - Footstep^{TM}

Exercise Benchmark

- Mannequin
  - 15 links, 20K triangles
- **Self-CCD performance**
  - **0.38 msec**

Motion Planning Benchmark 1

- Excavator
  - 52 links, 19K tri
- Obstacles
  - 0.4M tri

- **CCD performance**
  - **100~700 msec**

Motion Planning Benchmark 2

- Tower crane
  - 14 links, 1288 tri
- **CCD performance**
  - **5.66~15.1 msec**

Articulated Body Dynamics Benchmark

- Four trains
  - 10 links, 23K tri (each)
- **CCD performance**
  - **535 msec**

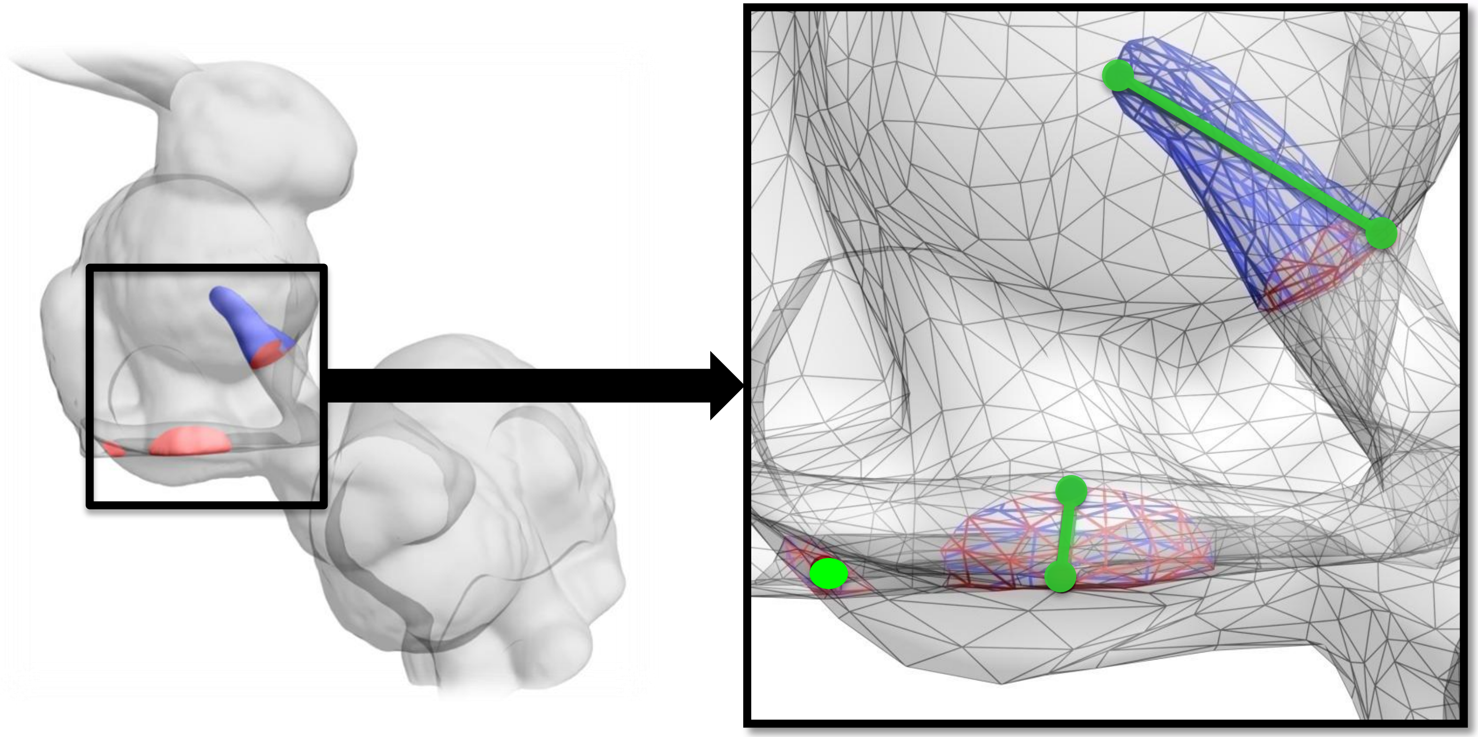
Software Implementations

- Source codes are available
  - http://graphics.ewha.ac.kr/FAST (2-manifold)
  - http://graphics.ewha.ac.kr/C2A (polygon-soups)
  - http://graphics.ewha.ac.kr/CATCH (articulated)

PENETRATION DEPTH (PD) COMPUTATION

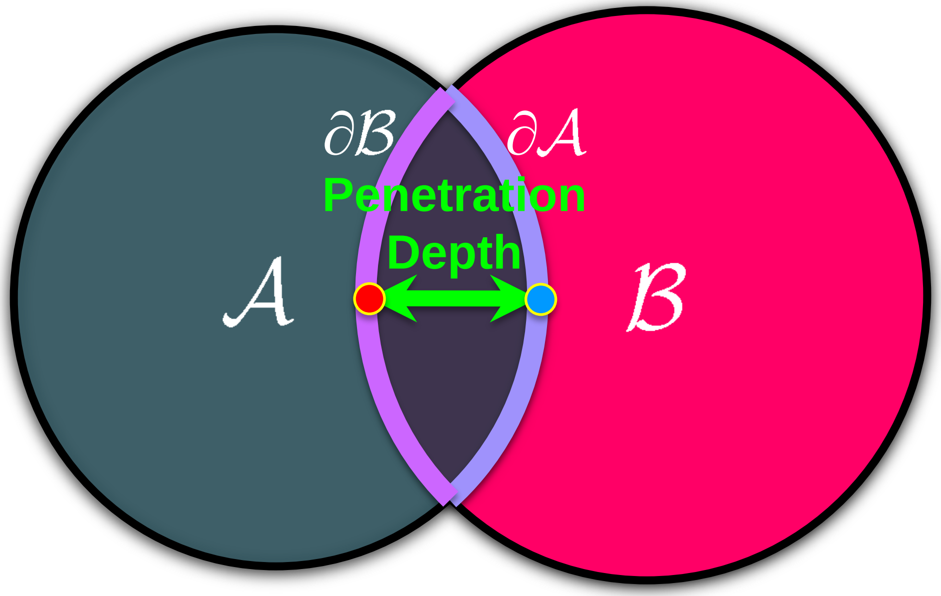
Pointwise Penetration Depth
[Tang et. al SIGGRAPH 09]

- Defined as deepest interpenetrating points



Pointwise Penetration Depth

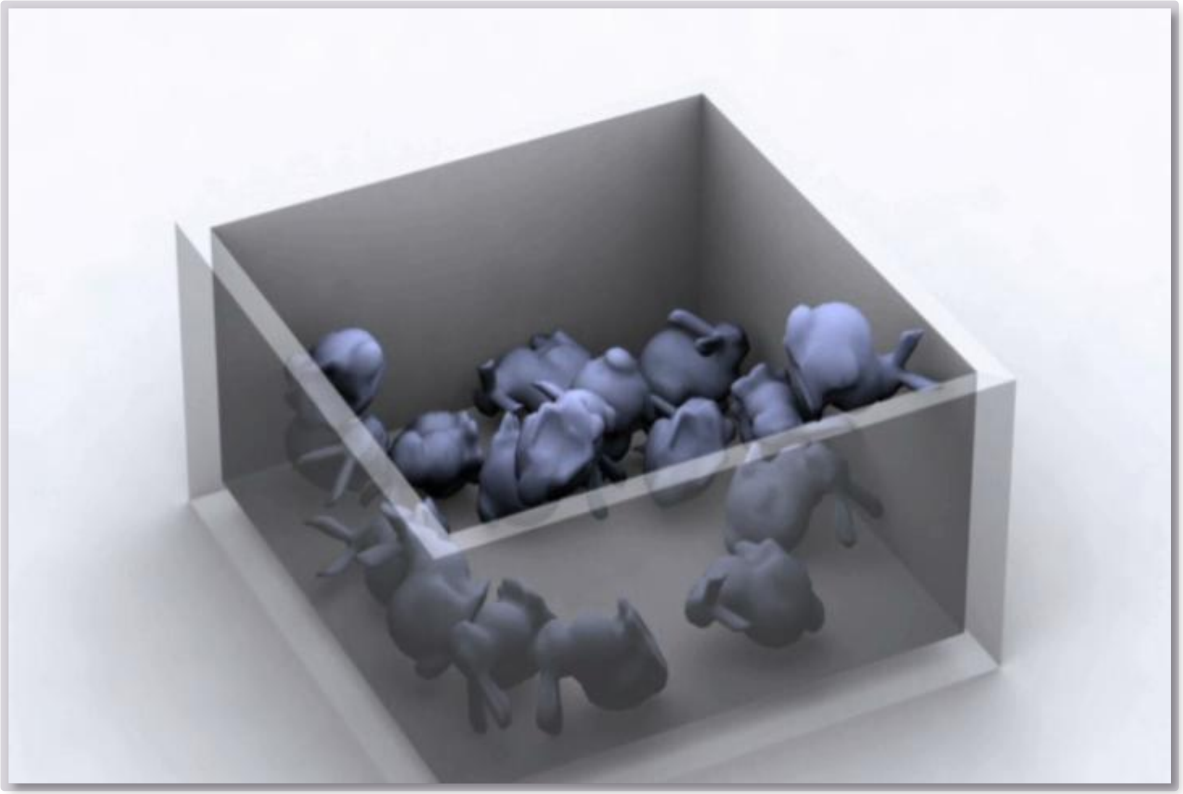
1. Find intersection surfaces $\partial\mathcal{A}$ and $\partial\mathcal{B}$
2. Penetration depth = $H(\partial\mathcal{A}, \partial\mathcal{B})$



Pointwise Penetration Depth

Demo (40K Bunny vs 40K Bunny)

Benchmark: Pointwise PD



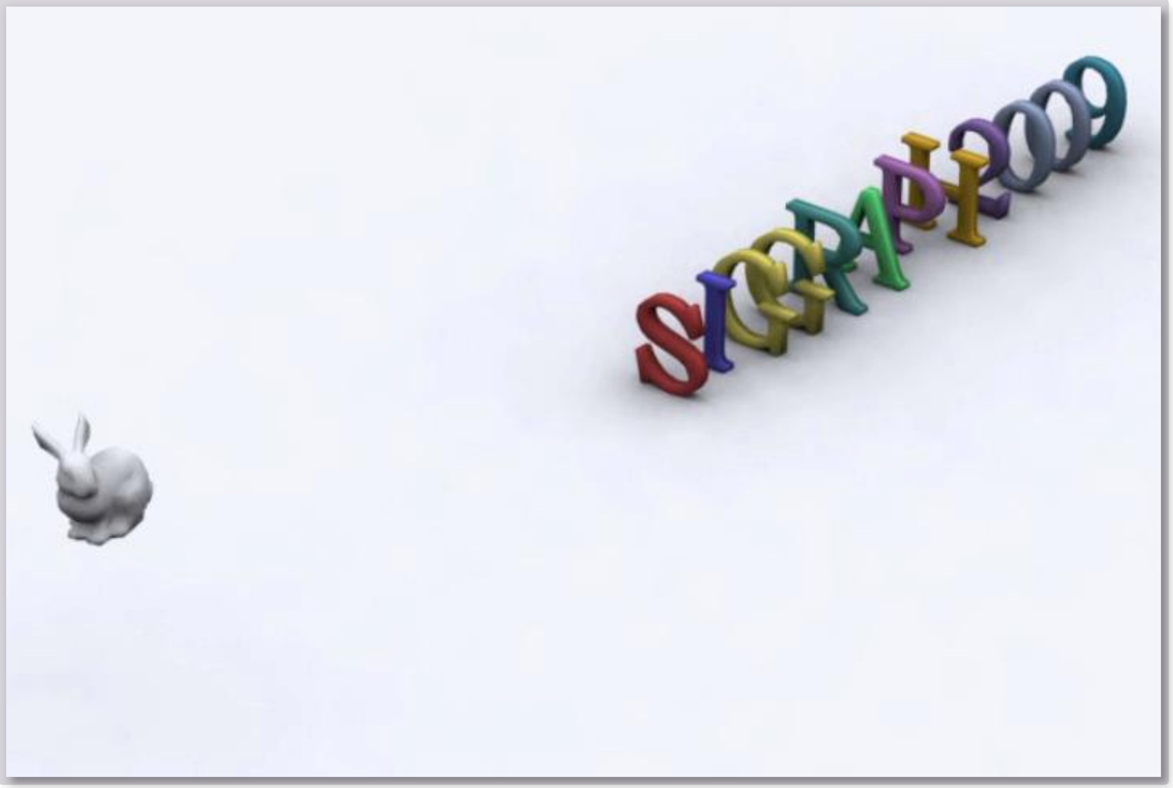
Model complexity

– 50K tri

Avg. Performance

– 3.88ms/pair

Benchmark: Pointwise PD



Model complexity

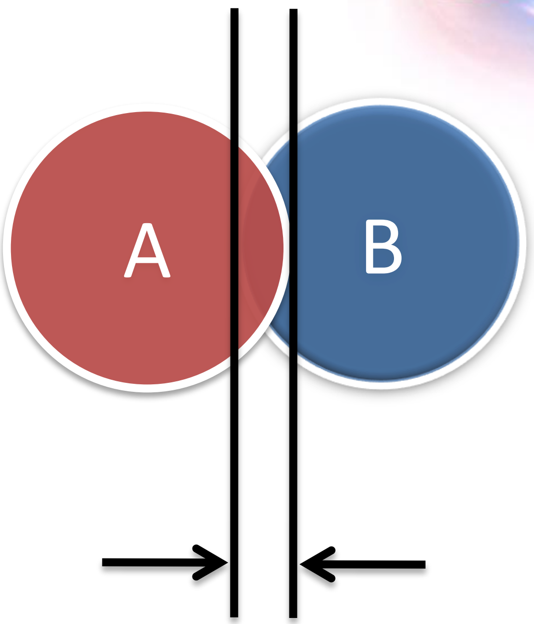
– 3.5K tri

Avg. performance

– 0.95ms/pair

Penetration Depth
[Dobkin 93]

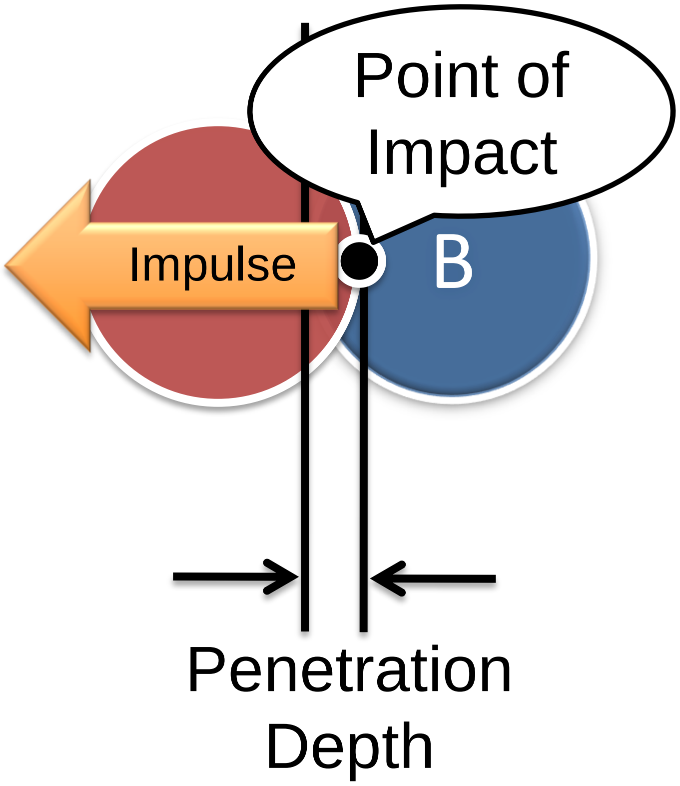
- Minimum translational distance to separate overlapping objects



Penetration Depth

Applications of Penetration Depth

- **Dynamics simulation**
  - **Penalty-based**
  - **Impulse-based**



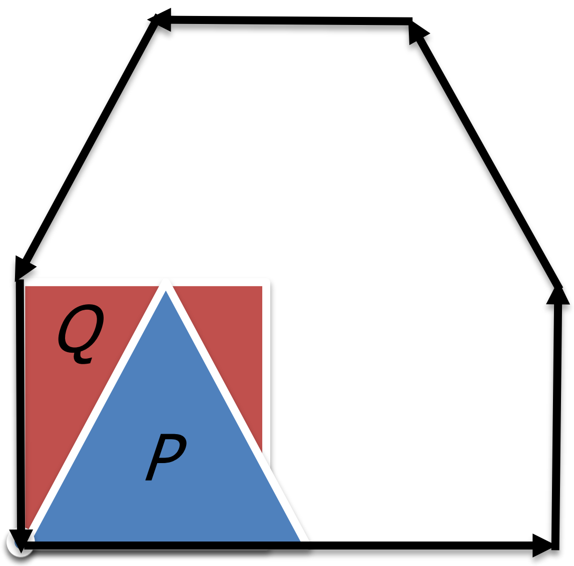
Previous Work on PD

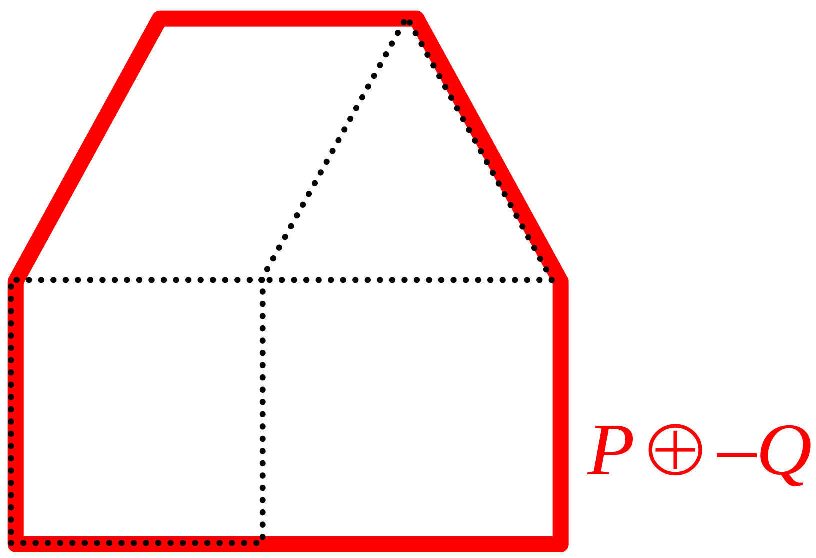
- Convex polytopes -[Cameron and Culley86], [Dobkin93], [Agarwal00], [Bergen01], [Kim04]
- Non-convex polyhedra -[Kim02],[Redon and Lin06], [Lien08a,b], [Hachenberger09]
- Distance fields -[Fisher and Lin01], [Hoff02], [Sud06]
- Pointwise PD -[Tang09]
- Generalized PD – [Ong and Gilbert96], [Ong96], [Zhang07]
- Volumetric PD - [Wellner and Zachmann09]

Minkowski Sum

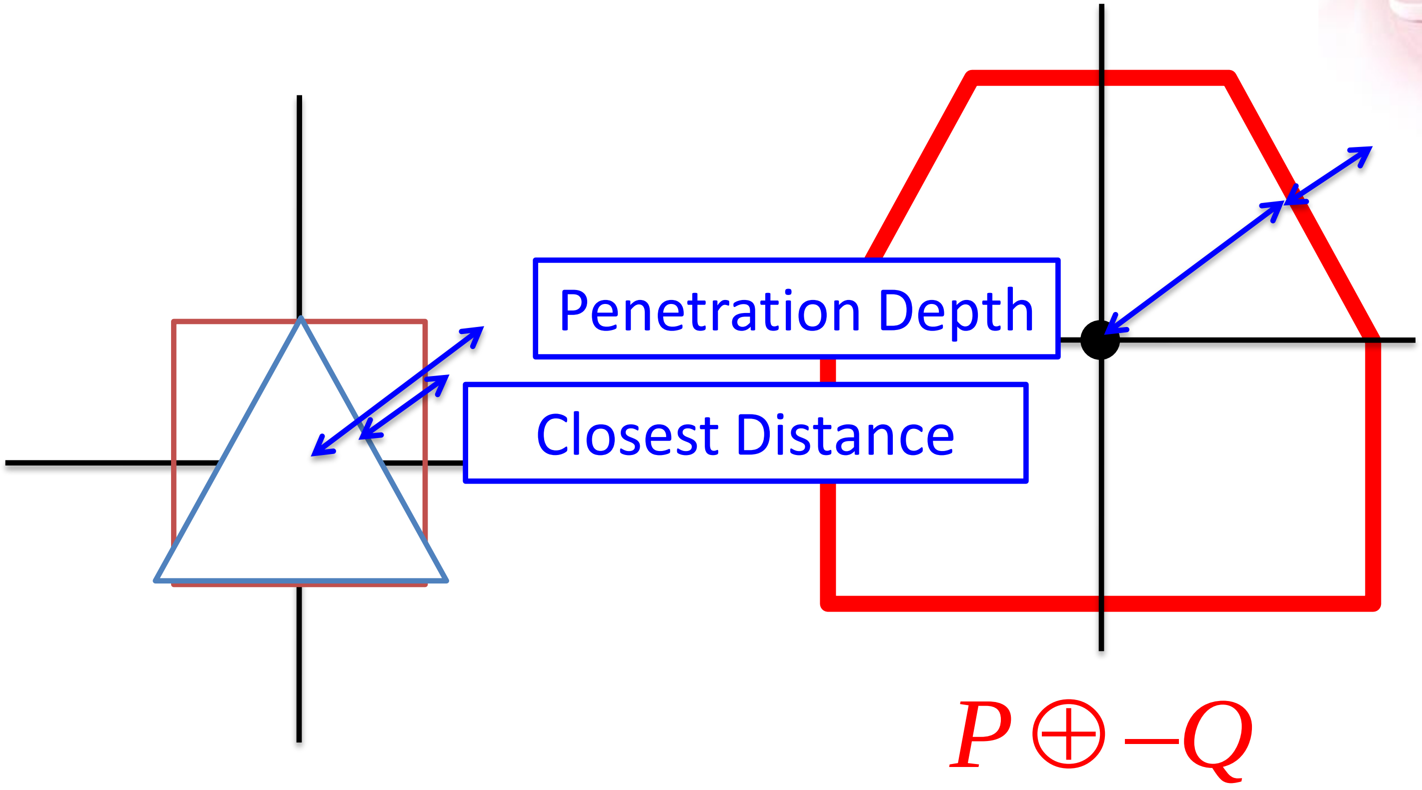
$$P \oplus Q = \{\mathbf{p} + \mathbf{q} \mid \mathbf{p} \in P, \mathbf{q} \in Q\}$$

$$P \oplus -Q = \{\mathbf{p} - \mathbf{q} \mid \mathbf{p} \in P, \mathbf{q} \in Q\}$$



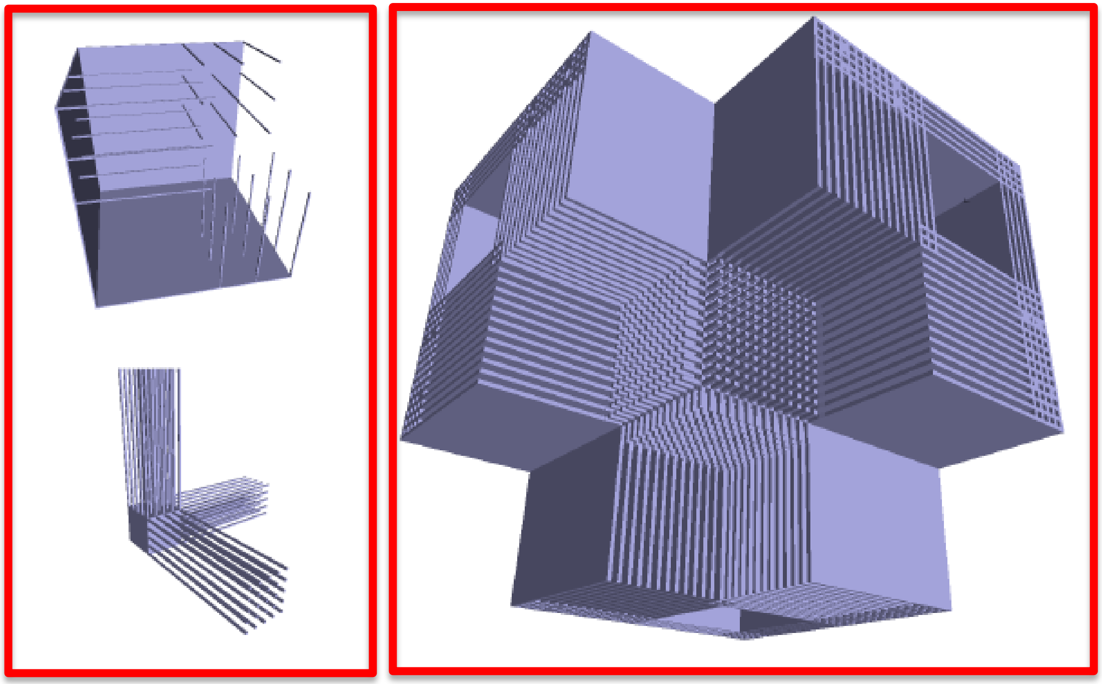


Proximity VS Minkowski Sum

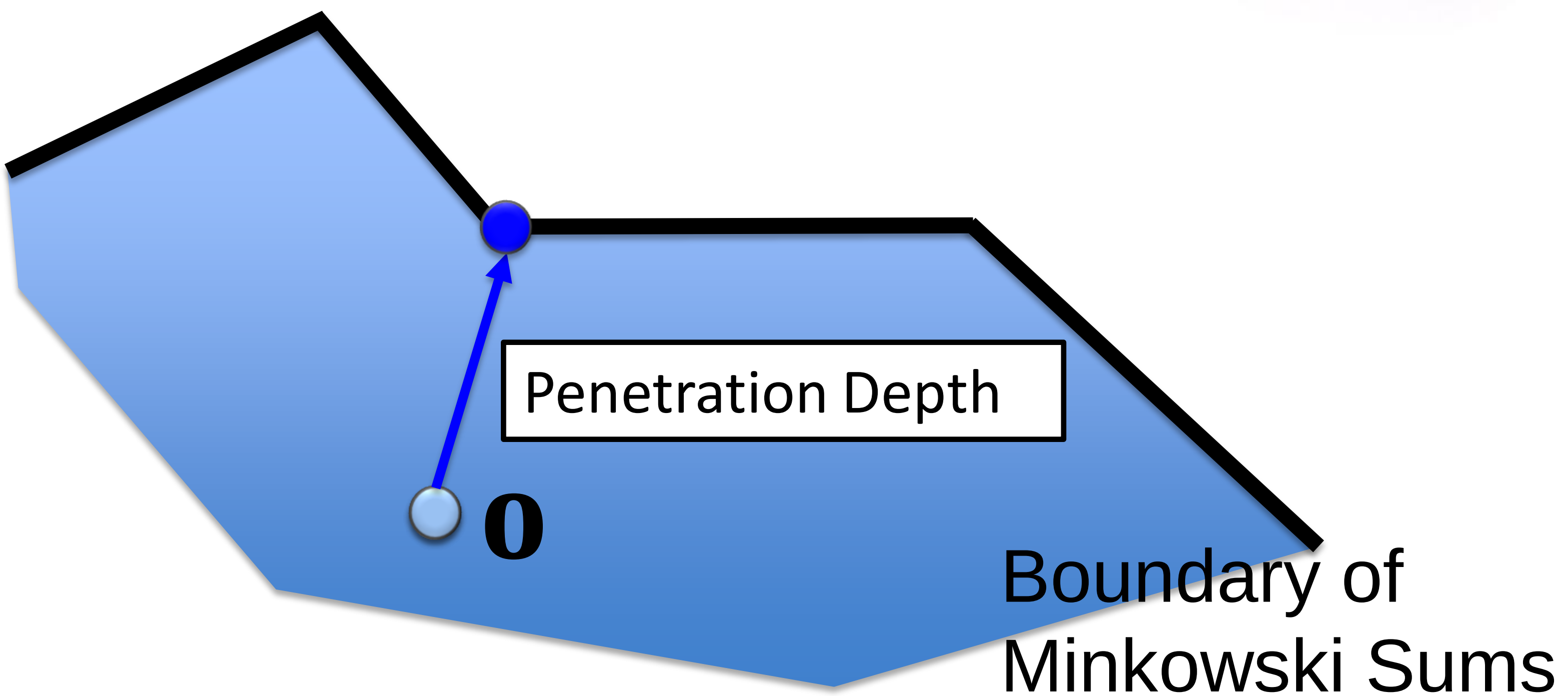


Combinatorial Explosion

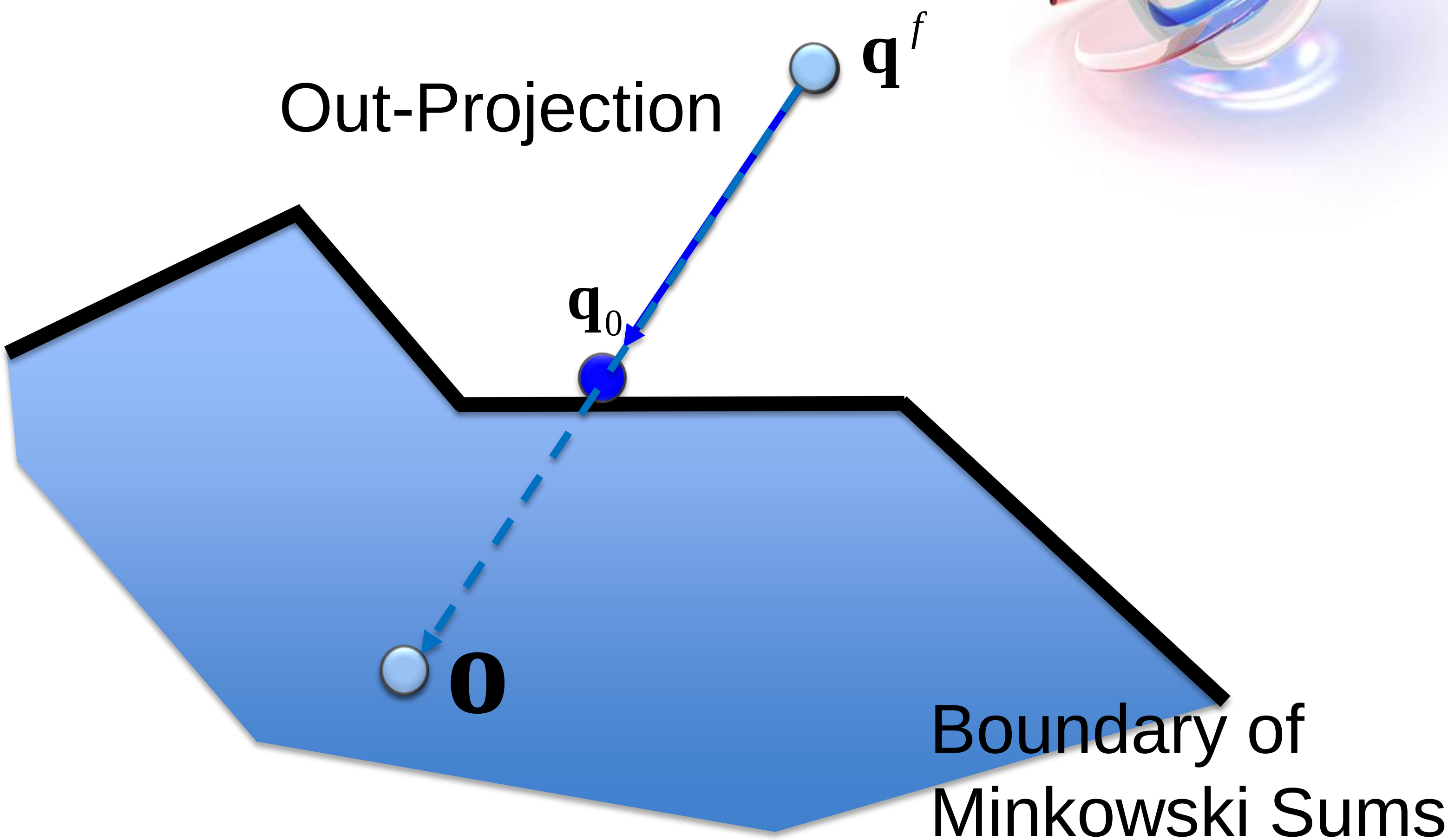
- Complexity of Minkowski Sum
  - $O(m^3n^3)$ with m and n triangles



PD Estimation
[Je et. al Tech Report 2010]

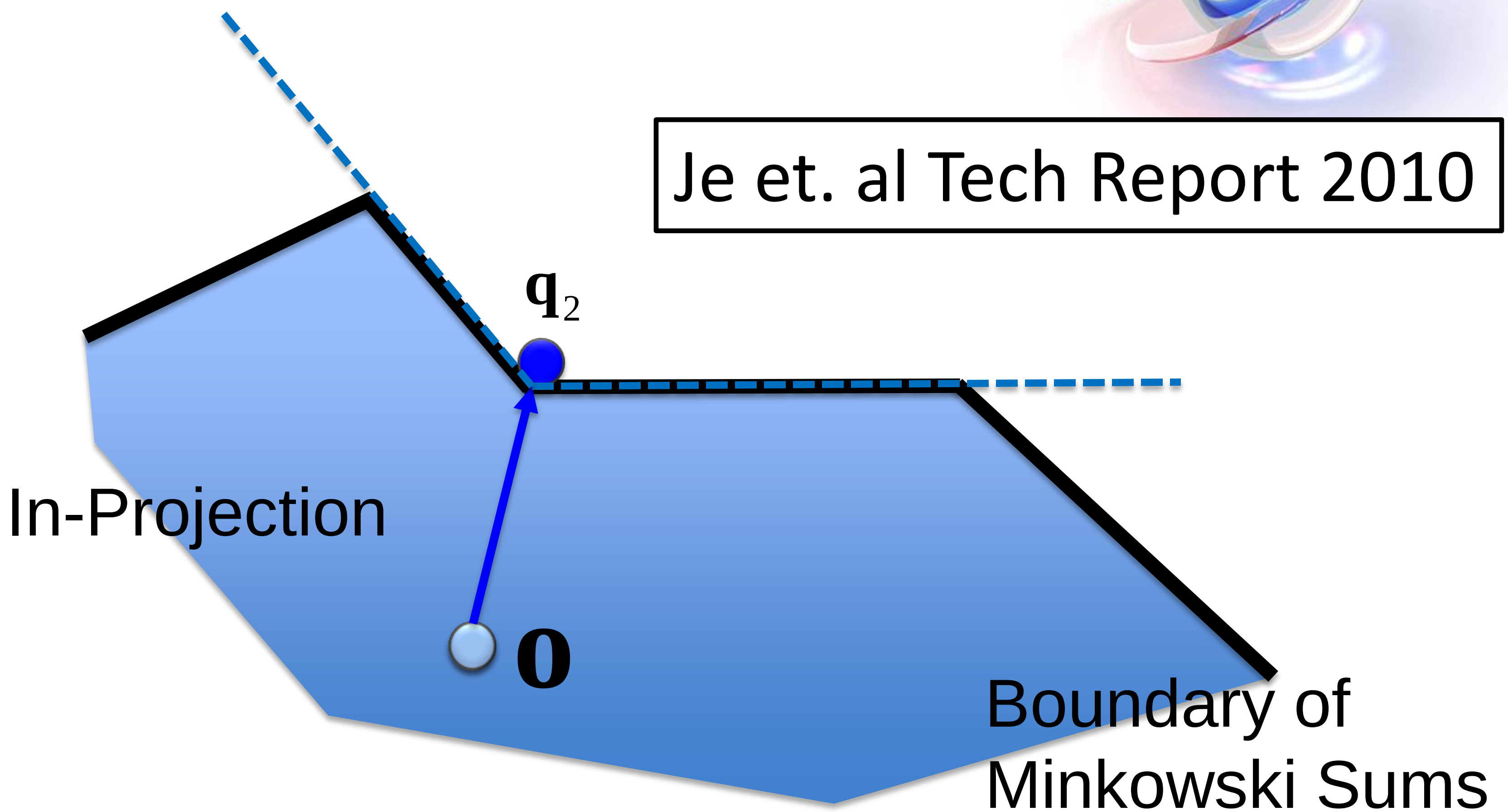


Out-Projection = CCD

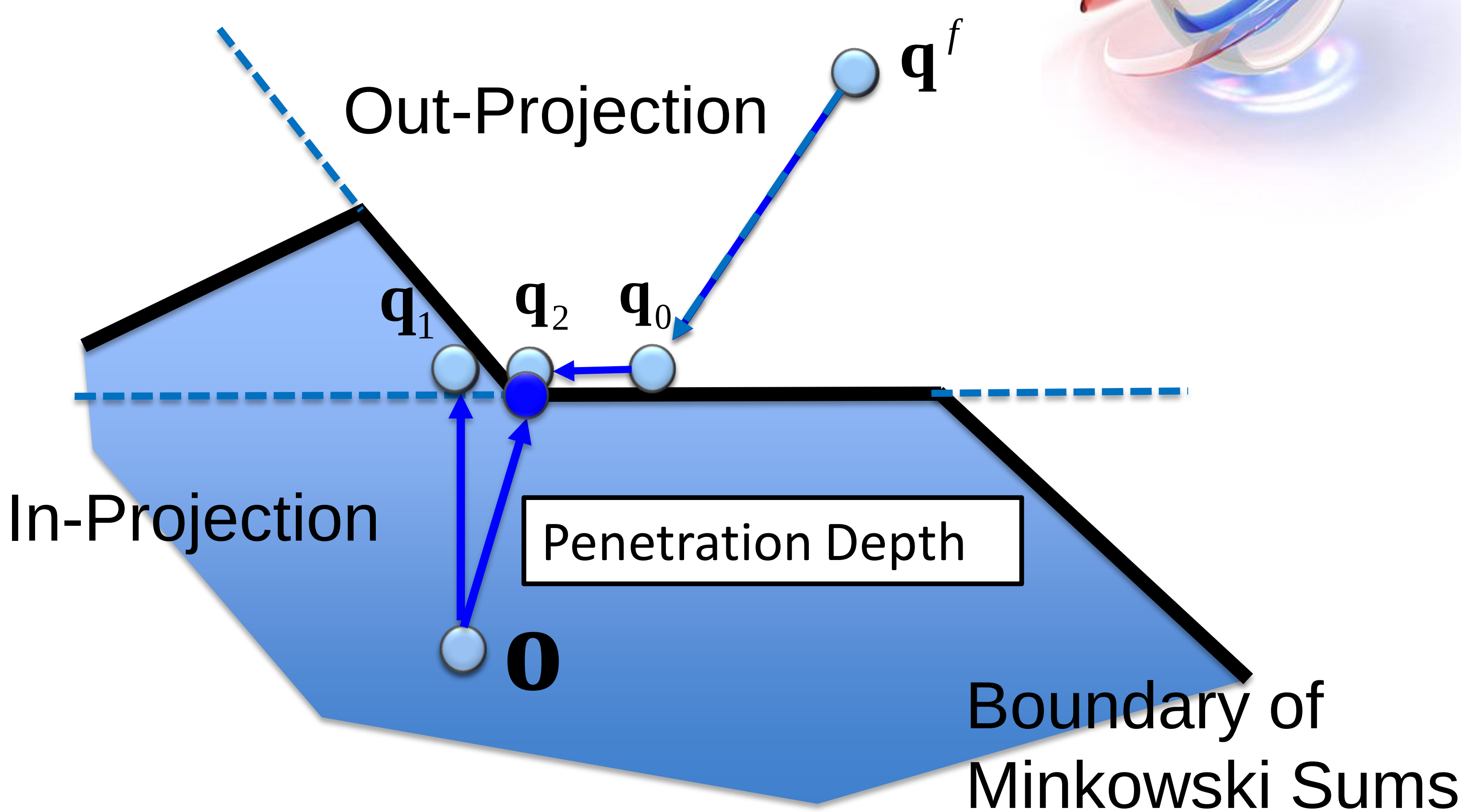


In-Projection = LCP
(Linear Complementarity Problem)

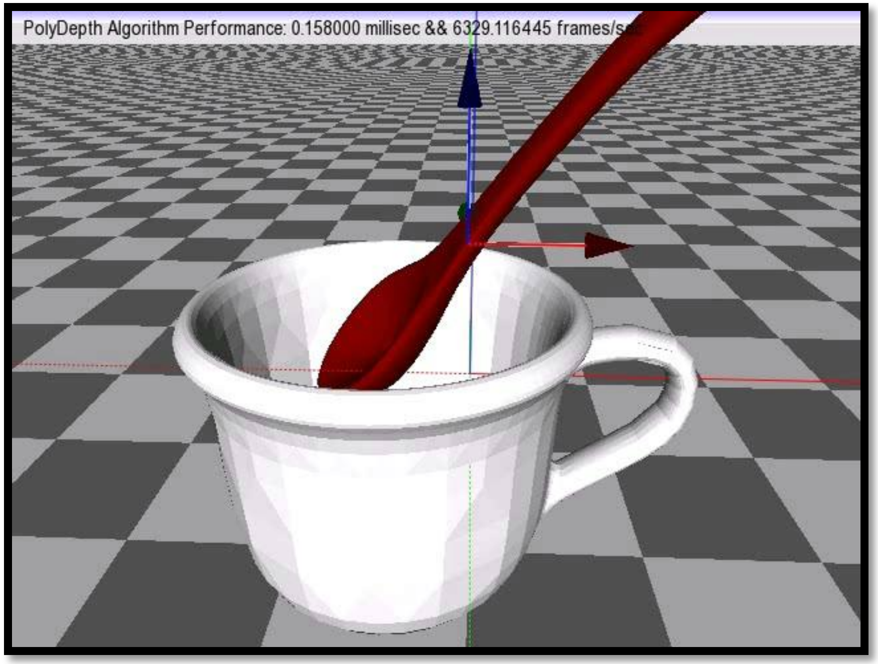
Je et. al Tech Report 2010



PolyDepth: Iterative Optimization



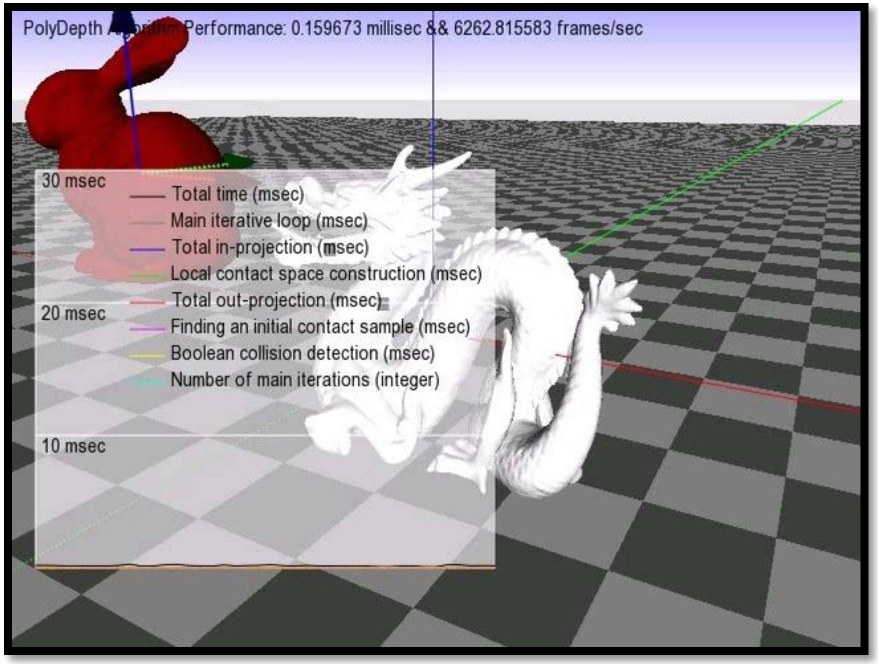
PolyDepth Performance



- Spoon: 1.3K triangles
- Cup: 8.4K triangles

- Time: 1~7 msec

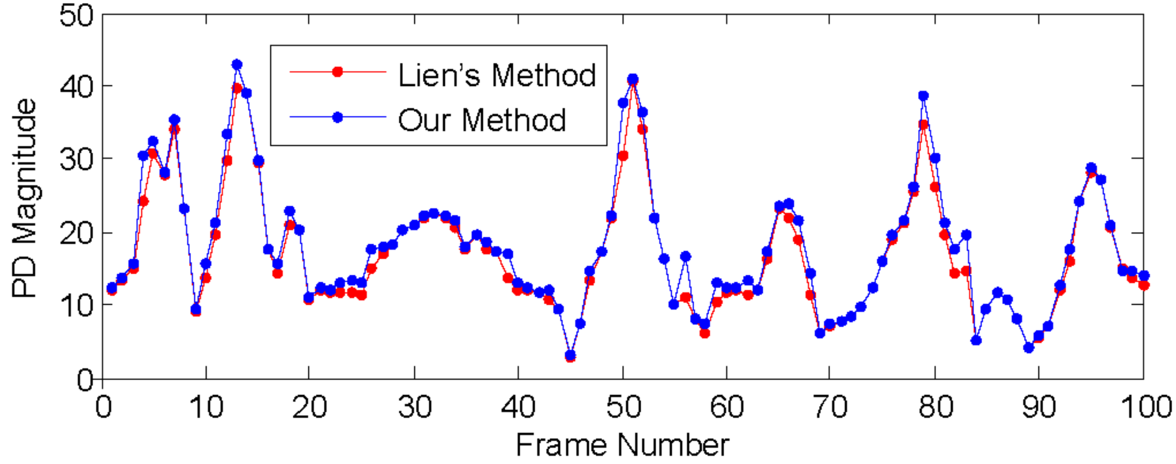
PolyDepth Performance



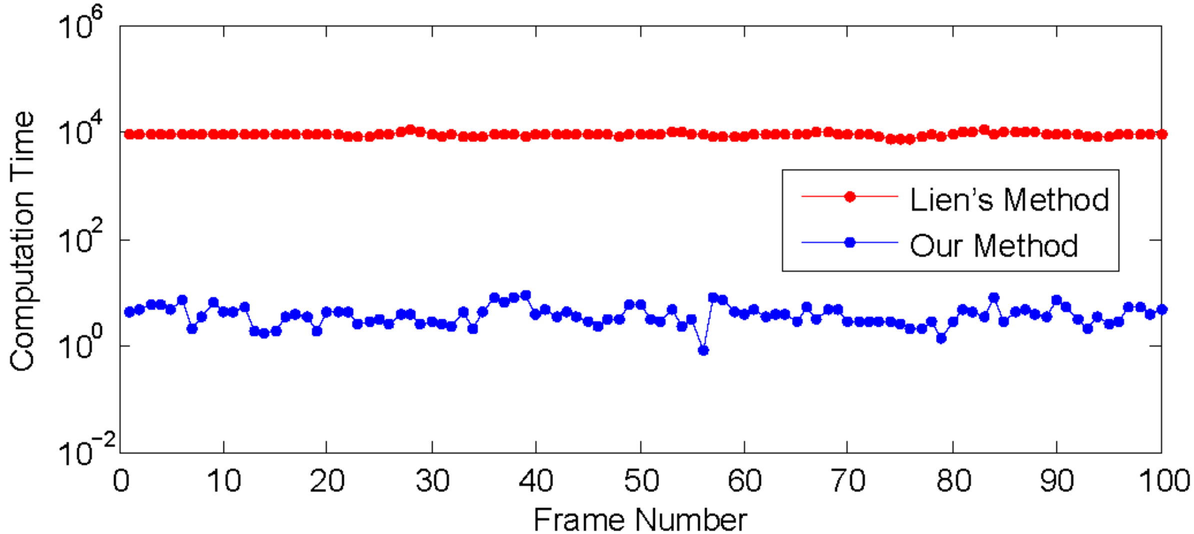
- Bunny: 40K triangles
- Dragon: 174K triangles

- Time: 2~15 msec

Comparison against Exact Solution



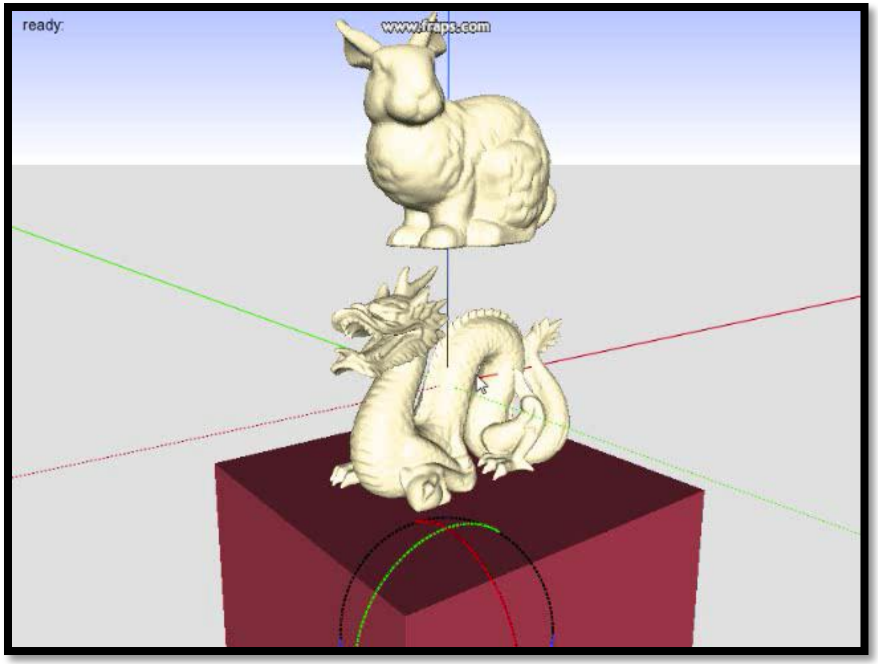
Accuracy

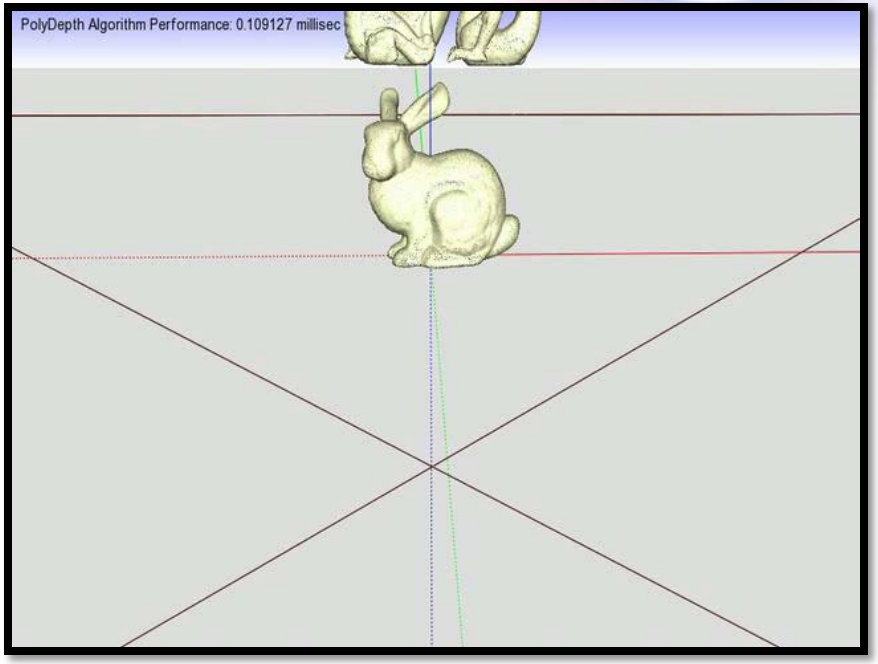


Performance

APPLICATIONS

Real-time Physics Simulation using PD

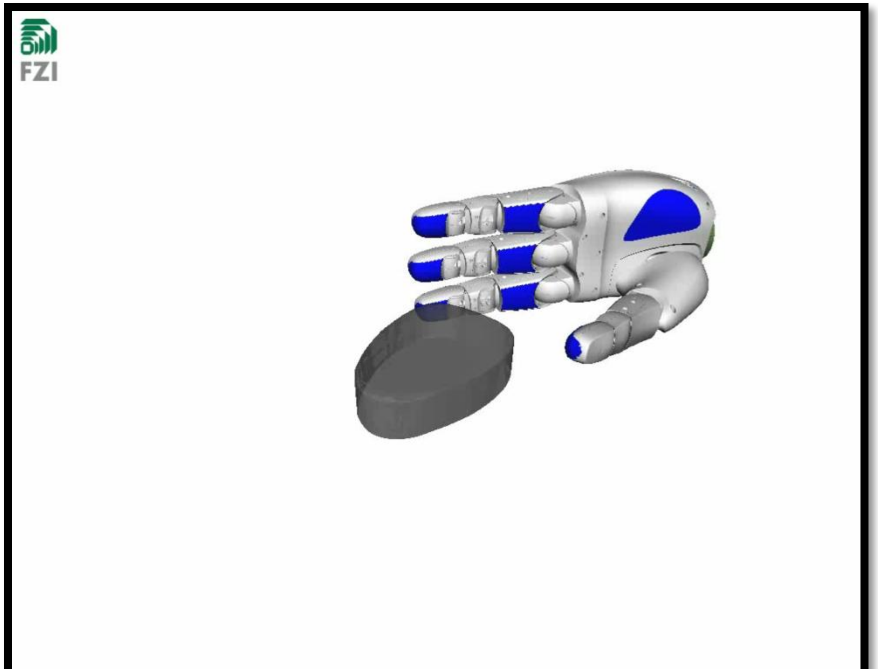




214K triangles in total

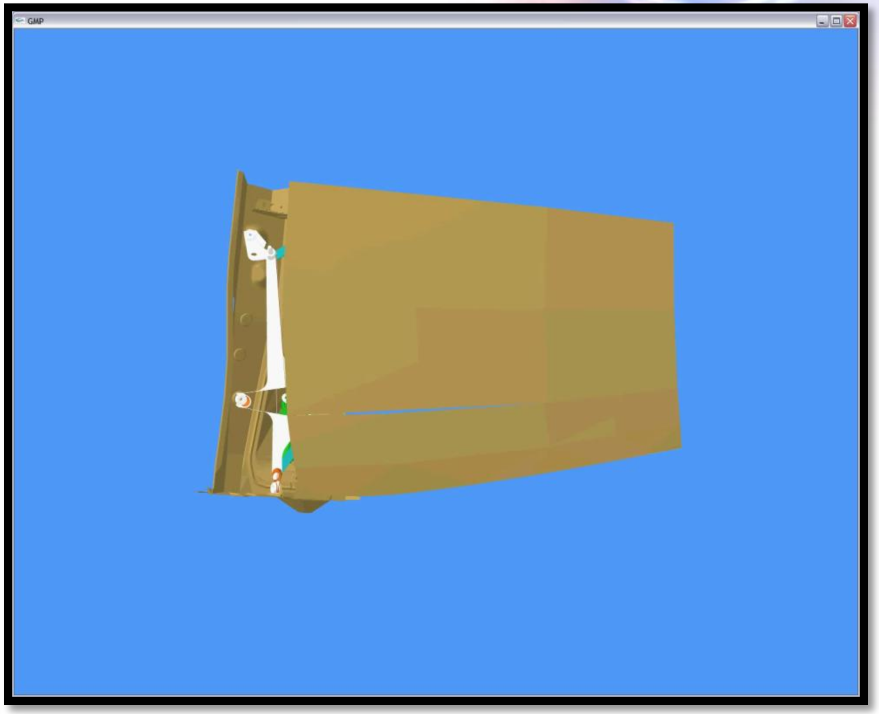
802K triangles in total

Robotic Grasping using CCD



With Zhixing Xue @ FZI

CAD Disassembly using CCD



With Liangjun Zhang @ Stanford/UNC

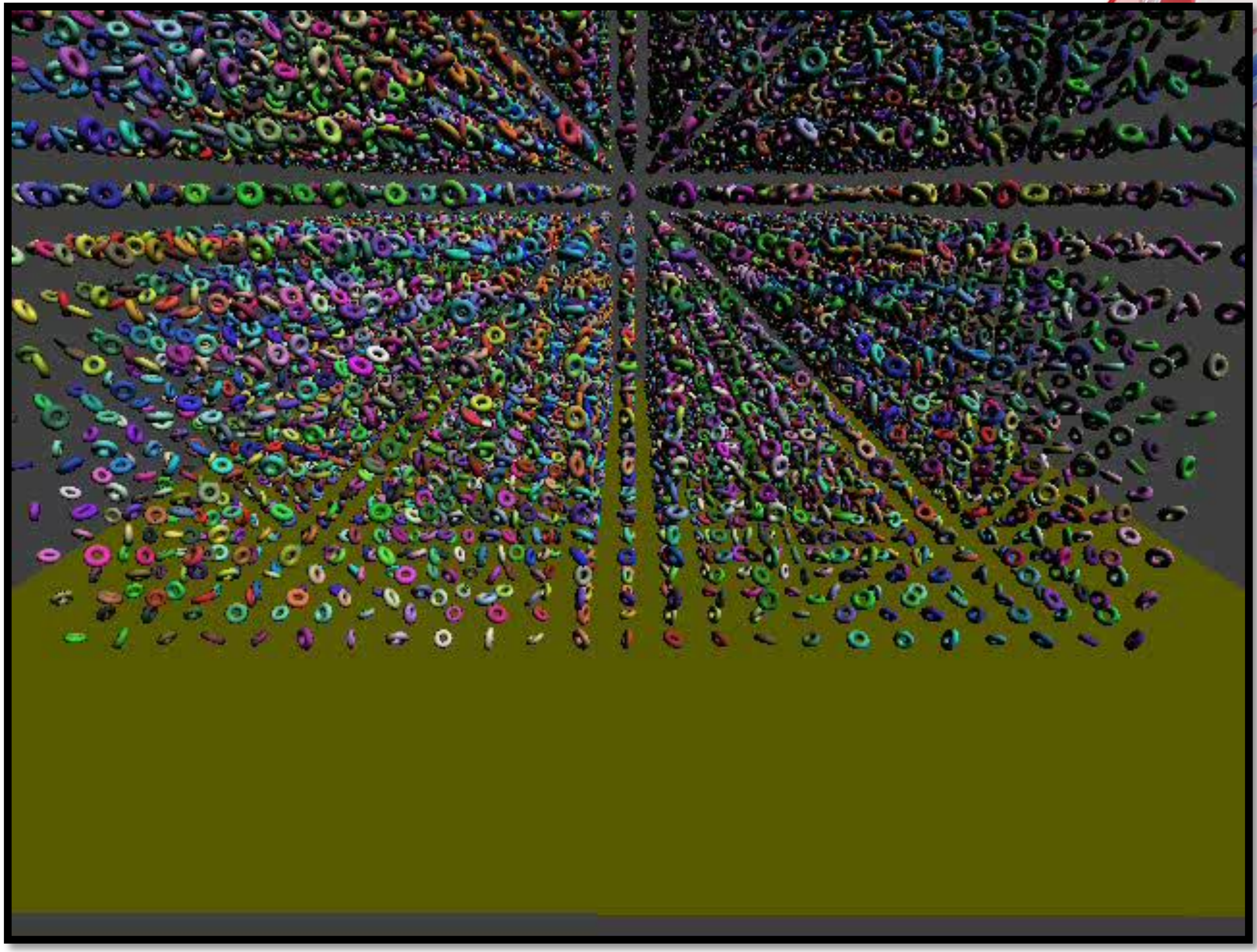
Summary

	CCD	PD
Concept	Collision avoidance	Collision correction
Usages	1. Constraint-based dynamics 2. Exact motion planning 3. Grasping	1. Penalty-, impulse-based dynamics 2. Retraction-based motion planning
Complexities	O(mn)	$O(m^3n^3)$

Future Work

- Continuous collision detection
  - N-body
  - Non-linear motion
- PD
  - Articulated body
  - Deformable
  - N-body

Collision Culling of a Millon Bodies on GPUs
[Liu et al. SIGGRAPH ASIA 2010]



Real-time Dynamics Simulation of 16,000 Rigid Bodies

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- Dinesh Manocha (UNC)
- Liangjun Zhang (Stanford)
- Zhixing Xue (FZI)

- KEIT/MKE (IT core research)
- KRF (Young investigator award)

Main References

- X. Zhang, M. Lee, Y. Kim, **Interactive Continuous Collision Detection for Non-convex Polyhedra**, Pacific Graphics 2006 http://graphics.ewha.ac.kr/FAST

- X. Zhang, S. Redon, M. Lee, Y. Kim, **Continuous Collision Detection for Articulated Models using Taylor Models and Temporal Culling**, SIGGRAPH 2007 http://graphics.ewha.ac.kr/CATCH

- M. Tang, Y. Kim, D. Manocha, **C^{2}A: Controlled Conservative Advancement for Interactive Continuous Collision Detection**, IEEE ICRA 2009 http://graphics.ewha.ac.kr/C2A

Main References

- M. Tang, M. Lee, Y. Kim, **Interactive Hausdorff Distance Computation for General Polygonal Models**, SIGGRAPH 2009
  http://graphics.ewha.ac.kr/HDIST

- C. Je, M. Tang, Y. Lee, M. Lee, Y. Kim, **PolyDepth: Real-time Penetration Depth Computation using Iterative Contact-space Projection**, Ewha Technical Report 2010
  http://graphics.ewha.ac.kr/PolyDepth

- M. Tang, Y. Kim, D. Manocha, **Efficient Local Planning using Connection Collision Query**, Ewha Technical Report 2010
  http://graphics.ewha.ac.kr/CCQ

Thank you for listening!

http://graphics.ewha.ac.kr